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INTRODUCTION TO ELECTRONIC ANALOGUE COMPUTERS

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EDITOR'S PREFACE

The aim of these monographs is to report upon research carried out in electronics and applied physics. Work in these fields continues to expand rapidly, and it is recognized that the collation and dissemination of information in a usable form is of the greatest importance to all those actively engaged in them. The monographs will be written by specialists in their own subjects, and the time required for publication will be kept to a minimum in order that these accounts of new work may be made quickly and widely available.

Wherever it is practicable the monographs will be kept short in length to enable all those interested in electronics to find the essentials necessary for their work in a condensed and concentrated form.

D. W. Fry

AUTHOR'S PREFACE

This monograph is based on the ideas and experience of a group of workers at the Royal Aircraft Establishment, Farnborough, Hants. This group was formed in 1945 under the leadership of W. R. Thomas, then at R.A.E., who adopted and developed computing techniques (Ref. 1) employed at the Telecommunications Research Establishment, Malvern (now part of the Radar Research Establishment) by a team which included F. C. Williams, F. J. U. Ritson, R. J. Lees, R. Aspinall and H. Sutcliffe. The R.A.E. group has grown continuously since that time, both in numbers of staff and number and complexity of computers, until it is now one of the largest groups of its kind in the country.

Substantial contributions to the work of this group have been made by: D. W. Allen, E. G. C. Burt, W. A. Elfers, J. J. Gait, O. H. Lange, F. R. J. Spearman, H. T. Ramsay, M. Squires, W. R. Thomas. The names of members of the group associated with particular developments are mentioned where appropriate, although references are not always given because the relevant reports have not been published.

Many inventions and improvements have been made by other workers, both in Britain and abroad, and acknowledgements are made on the text. I offer my apologies for any inadvertent omissions.

In the selection of material to fill the limited space available the aim has been to present basic principles rather than to describe detailed designs and design methods. Some acquaintance with the capabilities of electronic circuits and equipment is assumed, together with a mathematical background including simple differential equations.

I am particularly indebted to two of my friends and colleagues; J. J. Gait, for his careful and helpful reading of the manuscript and proofs, and K. C. Garner, who undertook the considerable task of preparing the diagrams and made a number of valuable suggestions.

Without sustained encouragement and assistance from my wife this monograph would not have been completed.

I have to thank the Chief Scientist, Ministry of Supply, for permission to publish this monograph. Figures 138 to 141 and 143 are Crown Copyright and are reproduced by permission of the Controller of Her Majesty's Stationery Office.

Farnborough, 1954 C. A. A. Wass

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ELECTRONIC CALCULATING MACHINES

In the past 10 or 15 years there has been a great growth of interest in calculating machines which depend for their operation on thermionic valves. During this period solutions have been required for the numerous complex mathematical problems which have arisen in the design of equipment required for military purposes, and there has been an increasing demand for the solution of difficult mathematical problems arising in the academic and commercial fields. Concurrently, electronic techniques have been developed which have permitted the construction of calculating machines of various kinds which can assist with the solution of these problems. None of these machines can perform mathematical operations other than those which a competent mathematician with paper and pencil can perform, but their capacity and speed is such that with their help it is practical to undertake the solution of problems which would involve prohibitive expenditure of labour by mathematicians and computors using the less costly and more familiar slide rule and desk calculating machine. The desk machine and also the mechanical differential analyzer based on the "ball and disc" integrator have, of course, been known for many years. The present work is concerned only with recent developments in machines and techniques which are wholly or largely electronic.

The electronic machines tend to fall into two broad classes, the "digital" machines and the "analogue" machines. It has been suggested by Hartree (Ref. 2) that the digital devices should be called "calculating machines" and the analogue devices should be called "calculating instruments", but this suggestion has not been widely supported, and in the present work both types will be called "machines".

The following paragraphs present the main features of electronic machines in such a way as to emphasize the differences between the digital and analogue types. The analogue machine is in no sense a replacement or substitute for the digital machine, and indeed it is rare to find problems which can be solved equally well by either type. The two types have features which fit them for different fields of application, and which make them attractive in different degrees to mathematicians, physicists and engineers.

1.1 DIGITAL MACHINES

The digital machines form a fairly well-defined class which has received some popular recognition under the name of "electronic brains", and they are usually fairly elaborate and costly devices, using a few hundred or a few thousand valves. They are basically arithmetic machines, the quantities they handle being represented as integral numbers of electrical pulses, and the fundamental operations which they can perform comprise addition, subtraction and discrimination, i.e. the determination of which of two given numbers is the larger. To perform more complex operations they must be provided with a "programme" of instructions in which multiplication, division, integration, etc., have been broken down into a series of additions and other simple operations. Continuous changes in the values of variables cannot be represented exactly, because the number of pulses representing a quantity cannot change by less than a single pulse; for calculations involving integration or differentiation the methods of the calculus of finite differences must be used. However, the effective number of pulses representing a quantity is often of the order of a million, so that a sufficiently close approximation to continuous variation is usually possible. This large number of pulses means also that high precision can be achieved, and even allowing some loss of accuracy from rounding-off errors, the results are often accurate to better than a part in 10^4 or 10^5, depending on the complexity of the problem.

The simple arithmetic operations can be carried out in a few microseconds, and more complicated operations, for example the calculation of the sine of an angle by summing a sufficient number of terms of a series, can be carried out in a small fraction of a second. Nevertheless, in computations which involve great numbers of steps the total time for a single computation may be as long as half an hour. For certain classes of problems digital machines are much slower than analogue machines (Ref. 3).

A feature of the digital machine is that it needs a central set of equipment for pulse generating, etc., which does not alter rapidly in size as the capacity of the machine changes, and whose size cannot usefully be reduced below a certain minimum. The machine cannot be smaller than this central equipment allows, and it is usually economical to make it considerably larger.

1.2 ANALOGUE MACHINES

The analogue machines cover a wider range of size than the digital machines, from simple "hook-ups" using perhaps only half a dozen valves to machines which are larger, both in number of valves and in physical size, than the large digital machines. The common feature of analogue machines is that the various quantities in the problem to be solved are represented by corresponding physical quantities in the machine. Thus, in the slide rule, which is one of the simplest analogue devices, the numbers in a problem are represented by lengths proportional to the logarithms of the numbers. Lengths on the rule are used as the analogue quantities whatever the nature of the quantities in the problem. In the electronic analogue machines the analogue quantities are commonly voltages which correspond in some predetermined manner with the quantities in the problem.

Analogue machines are generally much less accurate than digital machines. Errors as small as 0·1% are hard to achieve; errors of 1% are not unusual, and they are sometimes as large as 5%–10%. This is in striking contrast with the performance of the digital machines, but there are many problems where great precision is unnecessary. For example, in some aerodynamic calculations the aerodynamic derivatives may not be known to better than, say, 10%, and it would often be uneconomical to use more expensive equipment in order to reduce the errors in calculation to less than, say, one or two per cent.

The difference between the precision attainable by the two classes of machine arises from fundamental differences in the two methods of computation. In the digital machine the errors can be decreased without theoretical limit by increasing the number of pulses used to represent a quantity. In the analogue machine the total error is contributed by errors in the measurement of the physical quantities, such as voltage, and to variations in the characteristics of electronic components and valves; and although much can be done both to improve the accuracy of measurement and to reduce the effects of variations in characteristics the analogue machine cannot compete with the digital machine for calculations of the highest accuracy.

Analogue machines need no central set of equipment corresponding to the pulse-generating equipment of the digital machine, and it is economical and practicable to build quite small machines and extend these later if required.

Electronic analogue machines can perform addition, subtraction and some other operations directly, and can deal with continuously-varying quantities. In particular they can perform integration directly, provided that the independent variable is time. Methods have been devised for integrating with respect to other variables, but these have found little application, and for practical purposes there is no alternative to time as the independent variable.

This restriction on integration is a serious disadvantage of electronic analogue machines regarded as general-purpose calculators, when they are compared with digital machines or with mechanical differential analyzers based on the "ball and disc" integrator. Besides limiting the usefulness for solving ordinary differential equations this restriction makes it very difficult or impossible to solve partial differential equations. In the study of kinematic and dynamic systems, however, including aerodynamics and electrodynamics, this restriction is unimportant, and it is in this field that electronic analogue machines have found widest application. When the analogue machine is used in this way the variables and constants in the machine, and the way these quantities react on each other often present a close parallel with the behaviour of the actual system being studied, so that the machine is effectively a model of the system, i.e. a simulator. This feature often appeals to development engineers and experimental physicists, who are able to get a better "feel" of the problem in this way than if they have to present their problem in a formal manner for solution on a digital machine, probably via an intermediary mathematician.

An attractive facility of analogue machines is that parts of the actual dynamic system being studied may be included as part of the analogue calculating machine (Section 10.3). This is useful when the system contains some non-linear elements whose behaviour cannot be described in simple terms. If the dynamic system includes a man, say as a pilot or operator, the man can be included in the analogue computer, provided he can be presented with a satisfactory "display" of information.

The restrictions imposed by having no alternative to time as the independent variable are not quite as narrow or as complete as might at first be imagined. A differential equation containing derivatives of y with respect to x, where both y and x are independent of the time t, can sometimes be solved by replacing x temporarily by t, finding a solution for y in terms of t and substituting for t in the solution. Also, some problems in systems which are not normally regarded as dynamic systems can be solved by means of an analogue computer. A suggestion for solving problems in geometrical optics in this manner is made later (Section 10.1).

1.3 DIFFERENTIAL ANALYZERS AND SIMULATORS

The class of analogue machines which are wholly or mainly electronic, and which form the subject of the present work, can be subdivided in several different ways. In the next two chapters a distinction will be drawn between machines which are used as "differential analyzers" and machines which are used as "simulators". Formally, a differential analyzer is a device for solving differential equations, whereas a simulator may be defined for the present purpose as an electrical or electro-mechanical model of a dynamic system, so designed and arranged that measurement on the model gives useful information about the system. The difference between these definitions might suggest considerable differences between the two types of calculator, but in fact they have much in common. They use the same kinds of basic computing elements, and such differences as do exist are mostly in the way they are used, and in the attitude of mind of the user. This will become clearer in later chapters, but it can perhaps be illustrated by considering the different ways in which two men—a mathematician and an engineer—might attack some problem concerning a dynamic system. The mathematician would examine the system, write down the differential equations of motion, and then build—or have built for him—a differential analyzer to solve the equations. The engineer would examine the system and would then build a model of it, which he would call a simulator. He could then obtain solutions of his problem, perhaps without ever having written down the full set of equations of motion. Thus both men would obtain the required solutions, by somewhat different thought processes, but quite possibly the two calculating machines would be identical.

The term "simulator" is sometimes restricted to machines which work in "real" time, i.e. on a one-to-one time scale (see Section 10.1). This restriction will not be observed in the present work, and any machine which satisfies the foregoing definition will be referred to as a simulator, irrespective of the time scale in which it works.

1.4 ELEMENTS OF ELECTRONIC ANALOGUE COMPUTING

Electronic analogue machines generally use varying potential differences to represent the variables in the problem being studied and it will be assumed that this is so unless otherwise stated. The expression "voltage at a point" will be understood to mean the potential difference between the point and earth, measured in volts.

The voltages may be alternating, in which case the peak or R.M.S. values of a set of alternating voltages of constant frequency change in accordance with the variations of the variable quantities in the problem; or they may be what are loosely called "d.c. voltages", in which case it is the instantaneous value of each voltage which corresponds to some variable. In Britain the "d.c." machine is the more common, and in what follows this type will be assumed unless the contrary is stated.

The most important element of such machines is the high-gain directly-coupled amplifier, represented diagrammatically in fig. 1. Ideally, the output voltage V_O should be an exact magnified image of the input voltage V_1, whether V_1 is constant or varying, and the degree of magnification should approach infinity. In practical designs there is a reversal of phase through the amplifier so that V_1 and V_O are of opposite sign. The performance of the amplifier can therefore be represented by the equation

$$V_O = -MV_1 \tag{1}$$

where M is a very large positive constant. The shortcomings of practical amplifiers will be ignored for the present, and it will be assumed that amplifiers are available which obey equation (1) from "d.c." up to some frequency much higher than any frequency occurring in the problem, with values of M so high that they are effectively infinite. The input and output impedances of the amplifier proper will be taken to be respectively infinity and zero. It will further be assumed that the amplifier is single-ended (i.e. not push-pull) so that terminals C and D in fig. 1 are both connected to earth, and they will not be shown in later diagrams.

The most important applications of the high-gain d.c. amplifier are in the sign-reversing amplifier, the summing amplifier and the integrator. In principle some computations involving addition and

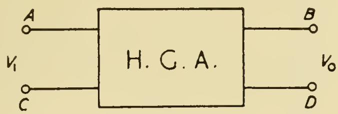


FIG. 1

integration can be performed without the need for high-gain amplifiers, but the loss in convenience and flexibility is so great that most analogue machines use amplifiers. For the present, attention will be confined to machines of this type, although some further reference will be made to this matter later (Chapter 6).

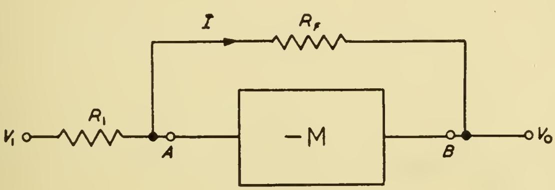


FIG. 2

The sign-reversing amplifier is simply a high-gain d.c. amplifier with two resistors, an "input" resistor and a "feedback" resistor, shown respectively as R_1 and R_F in fig. 2. Since the input impedance of the amplifier proper is infinite the net current at terminal A is zero, so that currents in R_1 and R_F are equal; i.e. if the voltage at A is V_A, then

$$\frac{V_1 - V_A}{R_1} - \frac{V_A - V_O}{R_F} = 0$$

Also,
$$V_O = -MV_A$$

Eliminating V_A,

$$R_F\left(V_1 + \frac{V_O}{M}\right) = -R_1\left(V_O + \frac{V_O}{M}\right)$$

$$\frac{V_O}{V_1} = -\frac{R_F M}{R_F + R_1(1+M)} \qquad (2)$$

And if M is very large,

$$\frac{V_O}{V_1} = -\frac{R_F}{R_1}, \text{ very nearly.}$$

In practice, if $R_F = R_1$, $V_O = -V_1$, and the output voltage is equal in magnitude to the input voltage, but of opposite sign. Obviously this arrangement, besides being used as a sign-reverser, can be used to multiply V_1 by a constant other than -1 if R_F and R_1 are given unequal values.

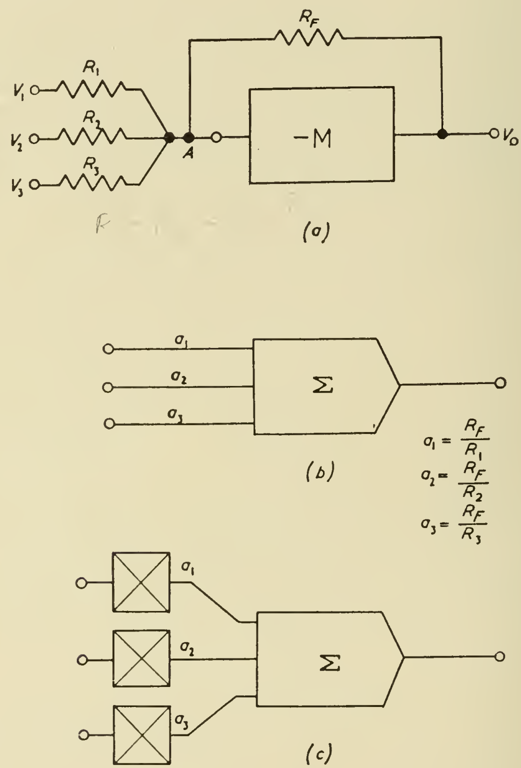


FIG. 3

The summing amplifier is similar to the sign-reversing amplifier except that there are several input resistors, as shown in fig. 3*a*. The net current at A is again zero, so that

$$\frac{V_1 - V_A}{R_1} + \frac{V_2 - V_A}{R_2} + \frac{V_3 - V_A}{R_3} + \frac{V_O - V_A}{R_F} = 0 \tag{3}$$

and
$$V_A = -\frac{V_O}{M}$$

Hence
$$\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3} = V_O\left\{-\frac{1}{R_F}-\frac{1}{M}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_F}\right)\right\} \tag{4}$$

If M is very large, then, very nearly,
$$V_O = -R_F\left\{\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right\} \tag{5}$$

or
$$V_O = -(a_1V_1+a_2V_2+a_3V_3)$$

where a_1, a_2, a_3 are positive constants equal to R_F/R_1, R_F/R_2, R_F/R_3, respectively.

If
$$R_1 = R_2 = R_3 = R_F,$$

then
$$V_O = -(V_1+V_2+V_3)$$

Thus the summing amplifier with equal resistors gives an output voltage which is the negative of the sum of the input voltages. If suitable unequal values of resistance are used each voltage can be multiplied by a constant before the addition takes place. In either case, there is no restriction to three input resistors; any number can be used, so that any number of voltages can be summed.

In the preparation of diagrams of machines containing summing amplifiers it is often convenient to be able to indicate the gain between each input terminal and the output terminal, i.e. the values of a_1, a_2, etc., without showing the resistors specifically. Two devices will be used for this purpose; in the first the value of the gain is written beside each input lead, and in the second the gain values are assumed to be always unity, and the coefficients are introduced by inserting "multiplier boxes" at points in the leads before they enter the amplifier. These devices are illustrated in figs. 3*b* and 3*c*, for both of which the gains are the same as for fig. 3*a*. The Σ sign indicates that the amplifier is being used for summing, and the pentagonal block includes the high-gain amplifier and the input and feedback impedances.

If, in equation (3), V_A is set equal to zero, then equation (5) appears at once. This is consistent with the assumption of an effectively infinite value for M, since if the amplifier has infinite gain but a finite output voltage then the input voltage must be zero. This device of assuming the voltage V_A to be zero is sometimes useful in making a quick estimate of the characteristics of a high-gain amplifier circuit.

The circuit of the integrator is shown in fig. 4*a*, which is similar to fig. 2 except that the feedback resistor R_F has been replaced by a feedback capacitor C. This arrangement is often called the "Miller"

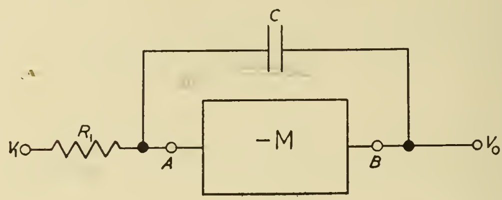


FIG. 4*a*

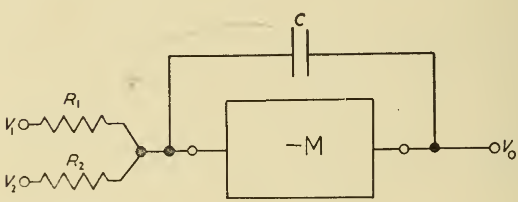


FIG. 4*b*

integrator, for a reason which will be mentioned later (Section 6.2). The current at A is zero, as before, and the current reaching A via the capacitor is equal to

$$C\frac{d}{dt}\left(V_O - V_A\right)$$

so that

$$\frac{V_1 - V_A}{R_1} + C\frac{d}{dt}\left(V_O - V_A\right) = 0$$

Again,

$$V_A = -\frac{V_O}{M}, \text{ giving}$$

$$V_1 + \frac{V_O}{M} + R_1 C\frac{d}{dt}\left(V_O + \frac{V_O}{M}\right) = 0 \qquad (6)$$

and if M is very large,

$$V_1 = -R_1 C \frac{dV_O}{dt}$$

Integrating,

$$V_O = -\frac{1}{R_1 C} \int_0^t V_1 dt + V_{OO}$$

where V_{OO} is the value of V_O at $t = 0$.

The question of initial conditions will be taken up later (Section 8.2), and for the moment it will be assumed that $V_{OO} = 0$, so that

$$V_O = -\frac{1}{T_1} \int_0^t V_1 dt \tag{7}$$

where $T_1 = R_1 C$ is called the "time-constant" of the integrator. Thus the arrangement of fig. 4*a* produces a voltage V_O proportional to the time-integral of an input voltage V_1.

An integrator can be used to provide the sum of the integrals or the integral of the sum of two or more quantities, without using a separate summing amplifier, by adding extra input resistors, as in fig. 4*b*. For this arrangement it is easily shown that

$$V_O = -\frac{1}{T_1} \int_0^t V_1 dt - \int_0^t \frac{1}{T_2} V_2 dt = -\int_0^t \left(\frac{V_1}{T_1} + \frac{V_2}{T_2} \right) dt$$

where $T_1 = R_1 C$, $T_2 = R_2 C$, and $V_{OO} = 0$.

When an integrator is shown in a block diagram the time constant may be indicated by writing the value inside the integrator block, or, alternatively, it is often convenient to assume that the time constant is unity and to perform the necessary level changes by means of "multiplier boxes" as for amplifiers.

By combining elements of these three types—reversing and summing amplifiers and integrators—in suitable ways, solutions can be obtained for linear differential equations with constant coefficients. The facility is not, of course, of great practical importance if the equations are of low order and only a few solutions are required, and if the disturbing function is of a simple form, such as a step function. The advantage of using such combinations is that large numbers of solutions, with different coefficients and different initial conditions, of equations of almost any order, can be obtained quickly and with comparatively little labour, and there is no restriction to simple disturbing functions.

It has already been mentioned that analogue computers tend to fall into two classes, *viz.* analyzers and simulators, and these will be treated separately in the following two chapters. The segregation of the two types of machine is convenient for the present purpose, and corresponds to very real differences, not only in the block diagrams, but also in the way the machines are set up and used. Nevertheless, it should be appreciated that the description of a particular machine as an analyzer does not necessarily mean that the machine is restricted to what will be called "analyzer methods", and similarly, machines which may be called simulators can usually be used as differential analyzers.

2

DIFFERENTIAL ANALYZERS

A differential analyzer is a device for solving differential equations. The name is not at all descriptive of the function of the device, and indeed, as VAN DER POL has pointed out (Ref. 4), the way in which a differential analyzer operates would make the title "integrating synthesizer" more appropriate. Furthermore, the description of the analyzer as a device for solving equations requires some qualification, because the manner in which the solutions are represented differs from the usual "paper" presentation in that the differential analyzer can produce only particular solutions to differential equations. There is no possibility of producing general solutions containing arbitrary constants which can be evaluated later from consideration of initial conditions; the initial conditions must be set into the machine before a solution can be obtained. In the study of dynamic systems the solution commonly takes the form of an indication of the manner in which some variable in the system changes its value with time in response to some stimulus, such as an impressed force. The stimulus must be specified exactly, though it may be of a formal character, such as a step or impulse function. In electronic differential analyzers the solution usually appears as a voltage varying with time, which can be fed into a suitable recorder to produce a graph.

There is, of course, no practical advantage in using a differential analyzer to solve simple equations, but it will help in understanding both the principles and some practical points if one or two simple equations with well-known solutions are considered first. Take, for example,

$$\frac{dx}{dt} - b = 0 \tag{8}$$

This may be regarded as a kinematic equation which says that the velocity of a particle (assumed to be moving in a straight line) is equal to b. The solution, whether it is obtained by analytical methods or by means of an analogue computer, will give x as a function of time, i.e. it will show how the position of the particle varies with time.

For solution of the differential equation by a differential analyzer the initial condition must be known, so assume for the present that $x=0$ when $t=0$. Equation (8) may be written;

$$x = \int_0^t b \, . \, dt$$

Equation (7), which represents the behaviour of the integrator of fig. 4*a*, is

$$V_O = -\frac{1}{T_1}\int_0^t V_1 \, . \, dt$$

Thus, ignoring the negative sign for the moment, if V_1 is made constant, to correspond with a constant value of b, then V_O will vary with time in the same manner as x. To establish a quantitative correspondence, suppose that V_1 is made proportional to b, and when b has a value of one foot per second V_1 has a value s_1 volts. Then $V_1=s_1b$, and s_1 is a "scale factor" having the dimensions of volts per foot/second. Suppose also that $V_O=-s_2x$, where s_2 is another scale factor, having the dimensions of volts per foot; the reason for the negative sign will appear below. The dimensions of s_1 and s_2 differ by the dimension of time, which is accounted for by the dimension of T_1, a time constant.

Substituting for V_1 and V_O,

$$s_2x = \frac{s_1}{T_1}\int_0^t b \, . \, dt$$

or, if

$$s_1 = s_2T_1,$$

$$x = \int_0^t b \, . \, dt$$

which is identical with the original equation.

Assuming a constant value of b, the procedure for solving equation (8) by means of the differential analyzer, which in this case consists only of the integrator of fig. 4*a*, is now straightforward.

A convenient way of satisfying the condition $x=0$ when $t=0$ is to assume that the velocity is zero for all negative time and increases instantaneously to the value b at $t=0$. This assumption requires that in the arrangement of fig. 4*a*

$$V_1 = 0, \; t<0$$
$$V_1 = s_1b, \; t\geqslant 0.$$

These conditions can easily be reproduced by means of a changeover switch which connects the V_1 terminal to earth until $t=0$ and then suddenly switches the connection to a battery of voltage s_1b, as shown in fig. 5. For $t<0$ there is no input to the amplifier, so $V_O=0$ also. At $t=0$, the integrator operates on V_1, and V_O changes in a manner representing x, thus giving the required solution. It is not strictly true to say that V_O is proportional to x, since the two quantities are of opposite sign. The negative sign would be less obvious if a scale factor s_2' of opposite sign were used, giving

$$V_O = s_2'x, \text{ where } s_2' = -s_2,$$

but this would not alter the fact that as x becomes more positive with increasing t, so V_O becomes more negative. It is generally more convenient to keep all the scale factors positive. If it is desired to produce a voltage $V_2=s_2x$, of the same sign as x, a reversing amplifier can be connected after the circuit of fig. 5.

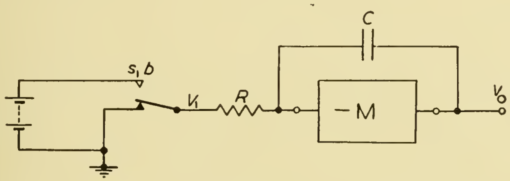


FIG. 5

The use of the integrator for solving equation (8) is of no practical value when b is constant, since the solution is already well known, but it may be useful when the value of b varies with time. Equation (7) shows that V_O is the time integral of V_1 (assuming $V_O=0$ at $t=0$ and again ignoring the minus sign) and this result is not dependent on V_1 being constant, so that V_1 may vary in any manner with time and V_O will always represent the time integral. Thus, if a particle begins to move in a straight line at $t=0$, then provided a voltage can be produced which is at all times proportional to the velocity of the particle, a second voltage can be produced by the integrator of fig. 4*a* which represents, at any instant, the position of the particle relative to its initial position. This is true whether the velocity of the particle is constant or varying, and independent of whether the variation is ordered or erratic. The last case is one which cannot be solved by a straightforward "paper" solution of equation (8) (page 13).

Although the assumption of a step function of velocity is a convenient and common means of satisfying the condition that $x=0$ at $t=0$ it is only acceptable if interest is confined to the period after $t=0$. The statement of the problem implies that the particle has been travelling along the straight line with velocity b for all time since $t=-\infty$ and that it will continue to do so until $t=+\infty$. However, for positive values of t there is no difference between the positions of this ideal particle and another particle which starts from rest at $t=0$ and then travels along the line with velocity b, provided the rest position of the second particle is at the point on the line through which the ideal particle passes at $t=0$. The identity between the two particles for positive values of t is sufficient justification for the use of the step function in practical problems. It is of interest to note that the circuit of fig. 5 can give a close parallel to the motion of the ideal particle, although of course infinite values of t and x cannot be accommodated. For suppose that the switch is in the upper position, but the capacitor has been charged to a high voltage, so that V_O has a value corresponding to a high negative value of x. This condition corresponds to some instant a long time before $t=0$, but the integrating action will proceed, and V_O will move nearer to zero and at some instant will pass through zero. If at this instant the clock measuring t is started, the conditions $V_O=0$ at $t=0$, and hence also $x=0$ at $t=0$, are satisfied. From $t=0$ the conditions are exactly the same as if the circuit had been quiescent, with the switch in the lower position, until $t=0$ and the switch had then been thrown to the upper position. If the clock were started at some other instant the value of x at $t=0$ would not be zero. This suggests a method of introducing a non-zero initial condition which will be discussed more fully in a later chapter (Section 8.2).

2.1 USE OF FEEDBACK

Consider now the equation

$$\frac{dx}{dt}+cx = e \tag{9}$$

and for the present assume that c is a constant and e is an input disturbance in the form of a step function. Assume also that $t=0$ when $x=0$. Here again the solution is well known, but solution by means of a differential analyzer introduces the important concept of feedback, which is necessary for all but the very simplest of differential equations.

It will be necessary to provide an input voltage V_1 to represent e, and the solution will appear in the form of a voltage V_O representing x. For simplicity assume that the same scale factor can be used for both these voltages, then

$$e = sV_1 \text{ and}$$
$$x = sV_O$$

These relations can be used to substitute for e and x in equation (9) giving the corresponding voltage equation

$$\frac{dV_O}{dt} + cV_O = V_1 \tag{10}$$

Remembering that the desired output is the voltage V_O it is reasonable to re-write this equation in the form

$$V_O = \frac{1}{c}V_1 - \frac{1}{c}\frac{dV_O}{dt}$$

Now the voltage V_1 is already available as the input voltage, and if there is also available a voltage V_2 proportional to dV_O/dt then sign-reversing and summing amplifiers can be arranged to combine the two voltages so as to produce V_O. The multiplication by $1/c$ can be achieved by adjusting the values of amplifier input and feedback resistors. The two voltages are to be subtracted, so it is necessary to reverse the sign of one of them before adding, and since a reversal will occur in the summing amplifier the V_1 voltage is passed through the reversing amplifier. A block diagram of the arrangement is

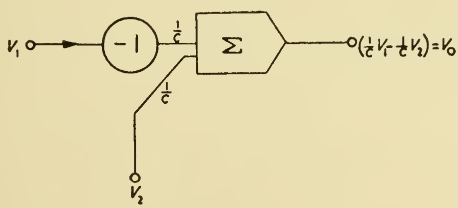


FIG. 6

shown in fig. 6, and this will provide the desired solution if means can be found to produce the voltage V_2. For simplicity it is assumed in the diagrams that the factor of proportionality between V_2 and dV_O/dt is unity. The circular block containing " -1" indicates a sign-reversing amplifier, including input and feedback resistors.

A computing element giving an output voltage proportional to the time-derivative of the input voltage, i.e. a differentiator, gives the desired V_2 when fed with V_O as input. Assuming that the differentiator, like some other computing elements, introduces a reversal of sign, the block diagram appears as fig. 7, in which the position of the reversing amplifier has been changed so as to compensate for the reversal in the differentiator. For reasons which will appear later the use of differentiators is avoided wherever possible, and they were not included among the basic computing elements described earlier. Nevertheless, although the arrangement of fig. 7 is not to be recommended as a practical analyzer it is of value because it introduces the idea of feedback loops, by which a voltage appearing at one point in the circuit is tapped off and reintroduced at some earlier point.

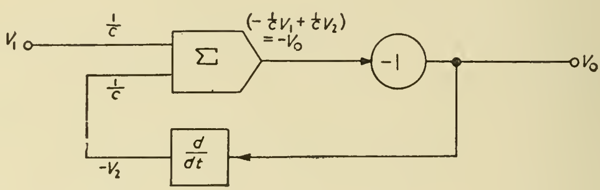


FIG. 7

The problem still remains of finding an arrangement for the solution of equation (9) which requires only amplifiers and integrators. A possible beginning is to assume that a voltage V_2 is available which is proportional to dV_O/dt, as before, but to use this as the input to an integrator which will give V_O as an output voltage. This will give the desired solution if V_2 can be provided, and this can be done by making use of the relation

$$V_2 = \frac{dV_O}{dt} = (V_1 - cV_O)$$

This leads to the block diagram shown in fig. 8. A reversing amplifier in the feedback connection gives the required negative sign for $-V_O$, and the coefficient c is introduced by using an input resistor of value $1/c$ relative to the feedback resistor in the summing amplifier. The pentagonal block with the integral sign ($\int$) represents an integrator, including high-gain amplifier, input resistor, and feedback capacitor. The reversal introduced by the summing amplifier is cancelled by the reversal in the integrator.

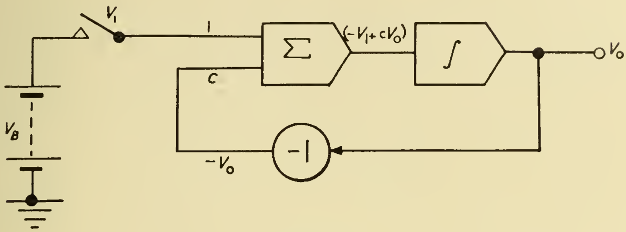


Fig. 8

In the arrangement of fig. 8 the dependence of the output voltage V_O on the input voltage is given by equation (10). If V_1 is varied with time in the same manner as e, then the consequent variation of x will be reproduced as variation of V_O and hence the equation will be solved.

On first acquaintance with closed loops such as that in fig. 8 doubt is sometimes felt as to the soundness of the arrangement, on the grounds that the solution cannot be computed until the answer is known, i.e. the second voltage required for the input to the summing amplifier cannot be produced until the output voltage from the integrator is available. Two considerations help to dispel this paradox. First, the usual procedure of describing the sequence of events in fig. 8 is sometimes taken to mean that there really is a sequence, in the sense that events happen one after another, with a small time interval between, say, the appearance of the two voltages at the input of the summing amplifier and the appearance of the sum voltage at the output. In fact, of course, with the present assumptions of ideal amplifiers, etc., there is no such interval, and the output voltages appear instantaneously. The second helpful point is that although it is certainly necessary for the voltage V_O to be present before the second input to the summing amplifier can be produced it is only the instantaneous value of V_O which is needed, and not the complete "answer".

The question is also sometimes raised as to whether such loops as fig. 8 are always stable. The answer is that if the analyzer loop is unstable then, assuming that the analyzer has been correctly set up, the dynamic system represented by the differential equation under consideration is also unstable.

Understanding of the mechanism of fig. 8 is improved by examining more closely the consequences of applying a step function by closing the switch. The voltage V_1 immediately rises to a steady value V_B, and since V_O is zero at this instant the output of the summing amplifier is $-V_B$. This voltage appears instantaneously when the switch is closed, and the integrator immediately begins to integrate. This means that the output voltage of the integrator begins to rise at the instant of closing the switch, so that the second input to the summing amplifier begins to increase from zero as soon as the switch is closed. The two input voltages are of opposite sign, so that the magnitude of the net input voltage to the integrator immediately begins to fall. The output of the integrator therefore continues to rise, but at a decreasing rate, and this continues until, after a theoretically infinite time interval, the output voltage of the analyzer reaches a value equal to V_B/c. Then the output voltage from the summing amplifier will be zero, and the integrating action will stop. Expressed more briefly, this means that a step function input voltage at V_1 will cause the output voltage of the integrator to rise exponentially towards a value proportional to the magnitude of the step.

The question of scale factors has been deliberately simplified in this example, but in practice the determination of a convenient and consistent set of values for scale factors, input and feedback resistors and integrator time constants is an important part of the setting-up procedure for an analogue computer, and more attention will be given to this topic later (Chapter 8).

2.2 A SECOND-ORDER PROBLEM

As a further example, consider the simple dynamic system of fig. 9. A mass m resting on a rough horizontal table, with coefficient

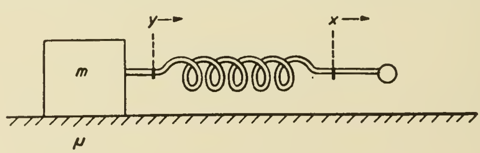


FIG. 9

of friction μ, is attached to a spring as shown. In the unstressed condition the turns of the spring are well separated, so that it can be both compressed and extended. When the mass is moving there is a frictional force proportional to the velocity. Let the positions of the free and attached ends of the spring be respectively x and y, relative to the initial positions, at which the spring is unstressed, and let the tension in the spring be $k(x-y)$. Then if the system is disturbed by displacing the free end of the spring horizontally there will be a force $k(x-y)$ acting on the mass due to the spring, and also a frictional force $-m\mu dy/dt$. The acceleration of the mass will therefore be

$$m\frac{d^2y}{dt^2} = k(x-y) - m\mu\frac{dy}{dt}$$

whence

$$\frac{d^2y}{dt^2} + \mu\frac{dy}{dt} + \frac{k}{m}y = \frac{k}{m}x \qquad (11)$$

A skeleton diagram of a differential analyzer for the solution of this equation is shown in fig. 10. No scale factors are shown, but the quantities represented by the voltages at various points are given. This arrangement is derived by first assuming that there is available a voltage representing the highest-order derivative, in this case d^2y/dt^2. Then voltages representing dy/dt and y can be obtained by the use of two integrators.

Re-writing the above equation in the form

$$\ddot{y} = \frac{k}{m}x - \mu\dot{y} - \frac{k}{m}y$$

where the dots indicate differentiation with respect to time, it appears that a voltage representing $\ddot{y}$ can be produced by combining voltages representing x, $\dot{y}$, and y. The x voltage is provided from some external source, and the $\dot{y}$ and y voltages from the integrator outputs. A sign-reversing amplifier is needed to give the appropriate sign for the $\dot{y}$ voltage, and the three voltages are combined by the

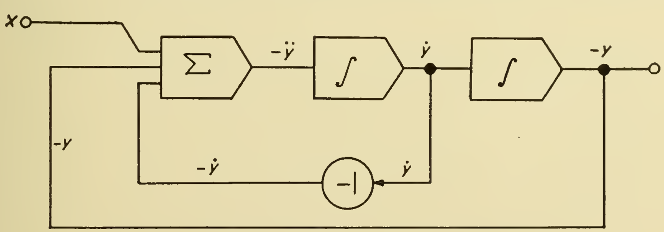


FIG. 10. Second-Order Differential Analyzer

summing amplifier whose output voltage, representing $\ddot{y}$, is used as the input signal to the first integrator.

Thus the output voltage of the second integrator in fig. 10 represents the variation in y corresponding to any variation x. As before, x can be the usual step function or any other function of time, provided that there is available a voltage proportional to x at all times.

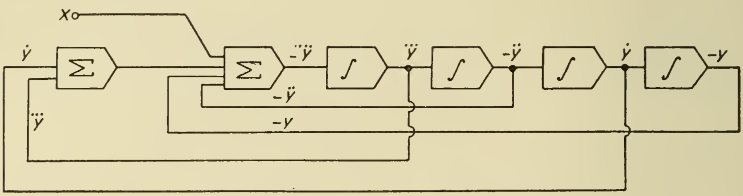


FIG. 11. Fourth-Order Differential Analyzer

The scheme of fig. 10 can be extended to solve higher-order equations, and in principle there is no limit. Fig. 11 shows an arrangement for solving

$$a_4\frac{d^4y}{dt^4}+a_3\frac{d^3y}{dt^3}+a_2\frac{d^2y}{dt^2}+a_1\frac{dy}{dt}+a_0y = x \tag{12}$$

Apart from the addition of two more integrators and their connections the only change is a second summing amplifier in place of the sign-reversing amplifier. Both dy/dt and d^3y/dt^3 must have their signs reversed before combining with x, and it would be satisfactory in principle to use two sign-reversing amplifiers. It is more economical, however, to use a single summing amplifier to change both signs.

In the foregoing examples, it was assumed that y and all the derivatives of y were zero at $t=0$. The method is not restricted to this set of initial conditions, and means for taking account of other initial values will be described later (Section 8.2).

2.3 DERIVATIVES OF THE INPUT QUANTITY

The examples have all referred to equations containing no function of x other than x itself. However, some equations which arise in the study of dynamic systems involve derivatives of x, and the solution of such equations by means of a differential analyzer raises a new difficulty. Methods are available which largely overcome this difficulty, but they are little used in practice because the "simulator" approach is generally preferred. Two examples will be given of dynamic systems which lead to equations which contain derivatives of x, but the analyzer method of solution for the second example is tedious and will be given in an appendix. The simulator method for both cases will be described in the next chapter.

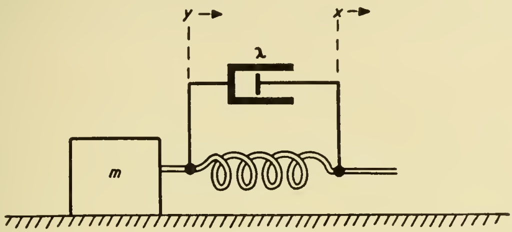


FIG. 12

As a simple example consider first the dynamic system shown in fig. 12. This is similar to that of fig. 9, but the table is now assumed smooth, and friction between the mass and the table is replaced by friction in the "dash-pot" connected between the ends of the spring. The frictional force, instead of being proportional to the velocity of the mass relative to the table is now proportional to the relative velocity of the ends of the spring, i.e. the frictional force is equal to

$$\lambda \frac{d(x-y)}{dt}$$

In place of equation (11) there now appears the equation

$$\frac{d^2y}{dt^2} + \frac{\lambda}{m}\frac{dy}{dt} + \frac{k}{m}y = \frac{k}{m}x + \frac{\lambda}{m}\frac{dx}{dt} \tag{13}$$

If a differentiator were available as a computing element the solution of this equation could be obtained by means of an analyzer

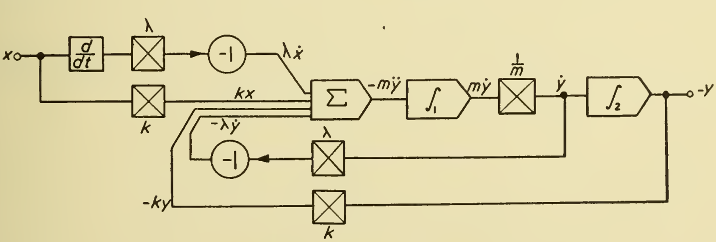


FIG. 13. Possible Analyzer for System of Fig. 12

as shown in fig. 13, which is similar to fig. 10, but has a differentiator added to produce the voltage representing dx/dt. It is assumed in this diagram that the differentiator gives a reversal of sign, so a sign-reversing amplifier is also added. In practice a second summing amplifier would be used in place of the two reversing amplifiers, the two voltages representing dx/dt and dy/dt being fed in through separate input resistors. The equation for the summing amplifier in fig. 13 is

$$m\ddot{y} = kx - ky + \lambda\dot{x} - \lambda\dot{y}$$

However, as already mentioned, it is inadvisable to use differentiators, and a method of solution is therefore required which uses only amplifiers and integrators. In this particular case such a method can be derived, and the first step in the derivation is to observe that the x voltage which is fed into the differentiator in fig. 13 is passed on, after differentiation, through a summing amplifier and then, together with other voltages, through an integrator. The two operations on the x voltage effectively cancel each other, and it is natural to enquire whether they can both be omitted. To test this suppose that the $\dot{x}$ voltage is disconnected from the input to the summing amplifier. This changes the output voltage to $(-m\ddot{y} + \lambda\dot{x})$, and the output of the first integrator becomes $(m\dot{y} - \lambda x)$. The deficiency λx can be removed by adding a voltage obtained directly

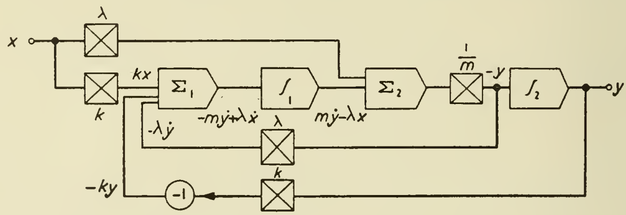


FIG. 14. Alternative Analyzer for System of Fig. 12

from the input voltage, as shown in fig. 14, where a second summing amplifier has been provided to perform the addition. There is now available a voltage proportional to $\dot{y}$, which provides one of the feedback voltages and also the input to the second integrator. The remainder of the circuit is unaltered except for the transfer of the reversing amplifier from one feedback line to the other, because of the reversal of signs of the $\dot{y}$ and y voltages. This method can be extended to higher-order equations, though as will be seen in the Appendix the procedure is not so simple if derivatives of x higher than the first are present.

Arrangements similar to that shown in fig. 14 can be derived by application of an interesting theorem due to BURNS (Ref. 5), although in the form he gives the method would be restricted to input disturbances which can be expressed in fairly compact mathematical form, whereas for electronic computers there need be no such restriction.

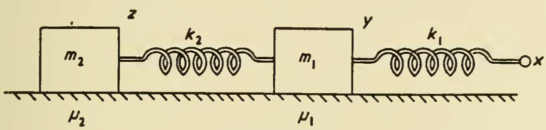


FIG. 15

A more complicated equation involving derivatives of the input quantity arises from the dynamic system shown in fig. 15. This is a "coupled" system, comprising two masses and two springs, and the assumptions are generally the same as for fig. 9. For the sake of generality unequal masses, friction coefficients, and spring constants are assumed.

The tensions in the springs of fig. 15 are:

$$T_1 = k_1(x-y)$$
$$T_2 = k_2(y-z)$$

and equating the total forces acting on m_1 to the product of mass and acceleration

$$m_1\ddot{y} = -m_1\mu_1\dot{y}+T_1-T_2$$

Substituting for T_1 and T_2,

$$m_1\ddot{y}+m_1\mu_1\dot{y}+k_1y+k_2y = k_1x+k_2z \tag{14}$$

For m_2,

$$m_2\ddot{z}+m_2\mu_2\dot{z}+k_2z = k_2y \tag{15}$$

From equations (14) and (15) y or z can be eliminated, giving expressions for z in terms of x, or for y in terms of x. These expressions involve fourth-order derivatives and for convenience here and subsequently use will be made of the operators

$$p = \frac{d}{dt} \qquad \frac{1}{p} = \int(\quad)dt$$

Eliminating z gives

$$ap^4y + bp^3y + cp^2y + dpy + ey = fp^2x + gpx + hx \tag{16}$$

where a, b, ... h, are functions of k_1, k_2, m_1, m_2, μ_1, μ_2 (see Appendix).

This equation has first- and second-order derivatives of x, and solution would be possible in principle by an analyzer similar to that shown in fig. 11 with the addition of two differentiators. However, in view of the objections to the use of differentiators such an arrangement is not practical, and for an analyzer solution one of the methods given in the Appendix must be used. A preferable alternative is to discard the "analyzer" approach and use the method given in the next chapter.

3

SIMPLE SIMULATORS

Simple simulators can be built up with the same basic elements as have been used so far in differential analyzers, and it is instructive to build up the block diagrams for simulators to solve the same problems as were used as examples for differential analyzers. This will show the difference in approach already mentioned, and although the block diagram for a simulator to represent a given simple dynamic system will be seen to differ little from the corresponding analyzer, there are considerable divergences when more complicated systems are considered. Some advantages of simulators, as compared with analyzers, will be demonstrated.

3.1 THE MASS-SPRING-FRICTION PROBLEM

The first problem concerns the mass and spring shown in fig. 9. The tension in the spring is equal to

$$T = k(x - y)$$

and the acceleration of the mass is related to the tension and frictional force by the equation

$$mp^2y = T - \mu mpy \tag{17}$$

The next step in the normal method of solution would be to eliminate T between these two equations. However, it is a feature of the "simulator method" that simulation of the equations should begin at as early a stage as possible, so that the problem involves the solution of a number of equations of low order rather than a single equation of higher order.

As a first step in setting up the block diagram, assume that there is available a voltage representing p^2y. If this is fed into two integrators in cascade two voltages representing $-py$ and y will be produced. The p^2y voltage can be produced by feeding into an amplifier voltages representing μmpy and T, and a suitable voltage for the first of these is available from the output of the first integrator. A voltage representing T can be produced by adding voltages representing kx and $-ky$ in a second summing amplifier, giving the block diagram shown in fig. 16. This arrangement is not very

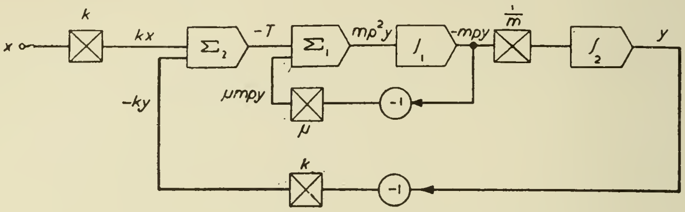


FIG. 16. Simulator for System of Fig. 9

different from that shown in fig. 10, and in fact if it is not required to observe the variation in tension T the second summing amplifier and the reversing amplifier in the feedback connection from the second integrator can both be removed, giving a diagram basically identical with fig. 10. However, although the block diagrams may be similar there is some difference in significance between the two arrangements. Thus, in fig. 16 the voltage fed back from the output of the first integrator now quite obviously represents the frictiona force, and the level of the feedback can be changed to represent different values of friction coefficient. This facility is, of course, available in the arrangement of fig. 10, but the connection between the physical effect and the corresponding voltage is somewhat less direct. The voltage representing spring tension does not appear at all in fig. 10, and although there would be no difficulty in providing it if it were required this would mean an extra step, whereas by the simulator approach it appears quite naturally.

For the system of fig. 12, with the friction force between the ends of the spring instead of between the mass and table, the first step in the application of the simulator method is to write down the equations in the simplest form, *viz.*,

$$T = k(x-y)$$
$$F = \lambda p(x-y)$$
$$mp^2y = F+T = \lambda p(x-y)+k(x-y)$$

where T is the tension in the spring and
F is the frictional force.

Assuming as usual that there is available a voltage representing mp^2y at the output of amplifier 1 in fig. 17, voltages representing *py* and *y* can be obtained by two integrators. (Here and elsewhere, for brevity, "amplifier" is used instead of "summing amplifier" when there is no likelihood of ambiguity.)

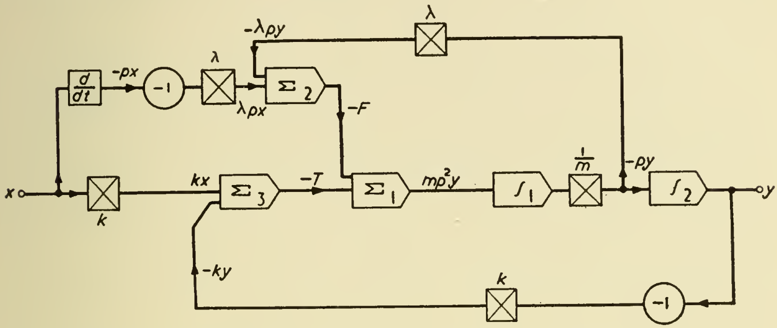


FIG. 17. Simulator for System of Fig. 12 using a Differentiator

According to the above equation the input voltages to the amplifier 1 must represent *F* and *T*, and assuming for the moment that a differentiator is available, these voltages can be obtained as shown, in accordance with the above equations for *F* and *T*. If it is not required to observe the forces *F* and *T* explicitly the arrangement can be simplified by performing all the additions in a single amplifier, and the arrangement then reverts to that of fig. 13.

The differentiator in fig. 17 can be removed and the deficiency made up as before by inserting an *x* voltage after the first integrator,

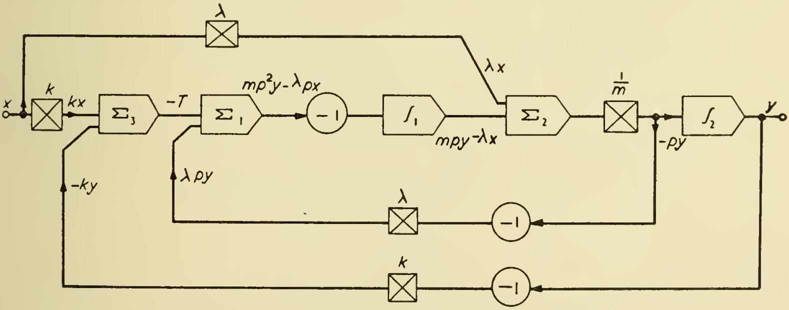


FIG. 18. Alternative Simulator for System of Fig. 12

as in fig. 18, which is similar to fig. 14 except that a voltage representing *T* is still available at the output of amplifier 3.

A further possible modification consists in disconnecting the *py* input of amplifier 1 (fig. 18), and correcting the deficiency by adding a voltage proportional to y after the first integrator giving the arrangement shown in fig. 19. Amplifier 3 of fig. 18 is no longer

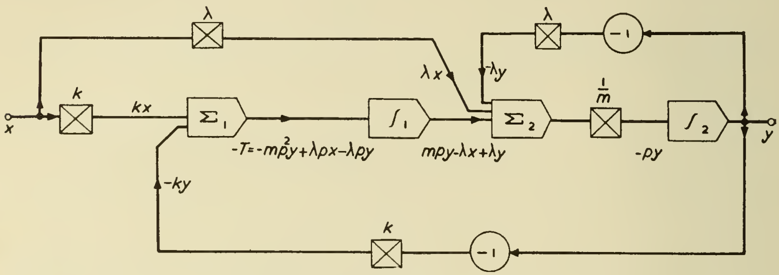


FIG. 19. Modification of Fig. 18

needed. If desired, the λx and $-\lambda y$ voltages could be combined in a separate summing amplifier before being fed into amplifier 2. The output of this additional amplifier would be proportional to F/p, which might be a useful quantity to observe. Also, since both the λ multipliers would be in the input leads to this amplifier it would only be necessary to vary the gain of this one amplifier to allow for changes in the value of λ.

The arrangements of figs. 18 and 19 do not provide such a complete model of the dynamic system as in the case of fig. 16 and the system of fig. 9. This is the penalty of avoiding the use of differentiators. Nevertheless, this simulator gives a slightly better insight into the dynamic problem than the analyzer of fig. 14.

If in the dynamic system of fig. 12 a rough table is substituted for the smooth table, so that two forms of friction are present, the appropriate change in the simulator can be made very easily. All that is necessary is to tap off a voltage representing $-mpy$, multiply by μ, and feed the resulting voltage, representing $-\mu mpy$, into the first summing amplifier, in either fig. 18 or fig. 19.

3.2 COUPLED MASS-SPRING-FRICTION SYSTEM

The mild advantages offered by the arrangement of figs. 16, 18 and 19 appear much more strongly when the simulator method is applied to the more complicated problem of fig. 15. The relevant equations are:

$$\text{Tension in spring 1} = T_1 = k_1(x-y) \tag{18a}$$

$$\text{Tension in spring 2} = T_2 = k_2(y-z) \tag{18b}$$

$$m_1p^2y = (T_1 - T_2) - \mu_1 m_1 py \tag{18c}$$

$$m_2p^2z = T_2 - \mu_2 m_2 pz \tag{18d}$$

In the "analyzer" treatment the tensions T_1 and T_2 were first eliminated, but for the simulator method the block diagram is built up immediately from these four equations. The procedure follows the same lines as before. Starting with equation (18*d*), which is identical in form with equation (17), assume that there is available, at the output of amplifier 2 in fig. 20 a voltage proportional to m_2p^2z. Proceeding as for fig. 16 leads to the arrangement shown in the lower part of fig. 20, which gives an output voltage representing z when the input is a voltage representing y. It is easily seen that the amplifiers 2 and 4 perform the additions indicated in equations (18*d*) and (18*b*).

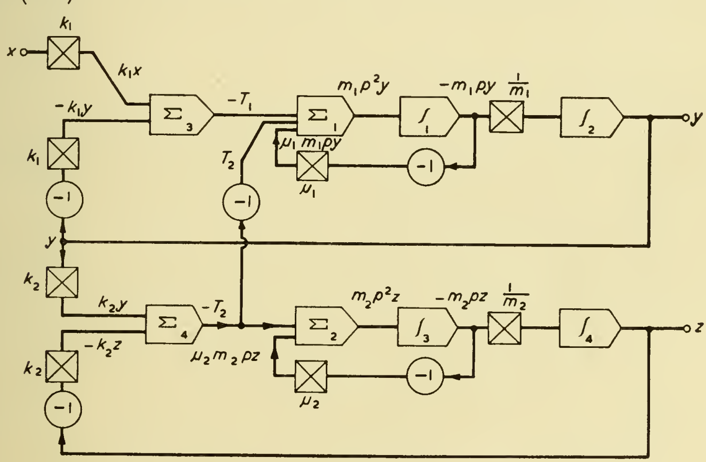


FIG. 20. Simulator for System of Fig. 15

For equation (18*c*) a similar procedure is followed, starting with the assumption that a voltage proportional to m_1p^2y is available, at the output of amplifier 1 (fig. 20). The inputs required for this amplifier include a voltage proportional to T_2, and this is obtained, as shown, from the output of amplifier 4 in the lower part of the diagram. Apart from this connection the two parts of the diagram are identical in form.

The arrangement of fig. 20 has a number of attractive features. The voltages correspond directly to physical quantities in the dynamic system, and changes in coefficients can easily be made. The simulator includes no derivatives higher than the second, and it gives simultaneous solutions for both y and z in a natural manner. In contrast, the form of equation (16) suggests strongly that solution by the "analyzer" method would involve representation of the third and fourth order derivatives, which have been introduced into equation (16) in a rather artificial way through elimination of z. Also, equation (16) has coefficients which are complicated functions of the masses, spring constants, and friction coefficients, which means that changes of these quantities would not be made so easily in the analyzer. Lastly, a separate analyzer, or at least a re-arrangement of the analyzer, would be necessary if solutions for both y and z were required. Besides these advantages fig. 20 gives an excellent picture of the system being studied. The lower part of the diagram represents a second-order system, similar to that of fig. 10, in which

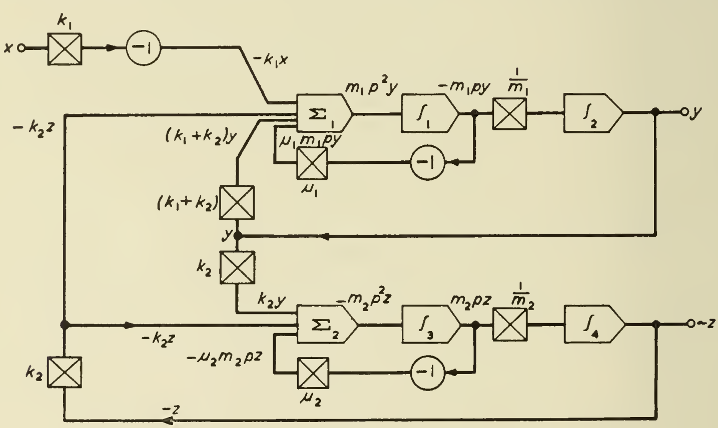


Fig. 21. Modified Form of Fig. 20

the only disturbance is the k_2y voltage coming from the upper part of the diagram. This corresponds with the left-hand part of the dynamic system (fig. 15), in which the second spring and mass constitute a second-order system in which the only disturbance is due to the movement of the end of the spring attached to mass m_1. Similarly, the upper part of the diagram represents again a self-contained second-order system, but here in addition to the disturbance due to x there is also the effect of the tension T_2, or $k_2(y-z)$, which corresponds exactly with the conditions in the real system. If it is not required to observe the tensions T_1 and T_2 explicitly the arrangement may be simplified, with some saving in amplifiers, by feeding the appropriate x, y and z voltages directly into amplifiers 1 and 2 instead of carrying out a preliminary combination in amplifiers 3 and 4 (fig. 21). The T_2 voltage feeding into amplifier 1 in fig. 20 is allowed for by changing the y voltage from k_1y to $(k_1+k_2)y$ and providing an additional input voltage to amplifier 1 representing k_2z. As a result of the omission of some amplifiers the signs of the quantities in the lower part of fig. 21 are reversed compared with fig. 20. The reversals can be cancelled by means of extra amplifiers if desired.

Although the system of fig. 20 or fig. 21 is capable of giving satisfactory solutions of the double mass and spring problem, it is of no great practical importance so long as all the coefficients are constant and x is some simple function of time, such as a step or impulse, or a sinusoidal variation. In such cases it would not generally be economic to build a simulator for this purpose only, since the required solutions can be obtained analytically. The attraction of a simulator increases, however, if the problem becomes more complex. Thus, suppose a third mass and spring is added to the system of fig. 15. This raises the corresponding differential equation to the sixth order, but the extension of the simulator of fig. 20 to include the third mass and spring involves no new principle. If the mass, spring factor, and friction coefficient are m_3, k_3, μ_3, and the position of the mass from its initial position is w, the tension in the third spring is

$$T_3 = k_3(z-w)$$

and for the third mass,

$$m_3p^2w = -\mu_3m_3pw+T_3$$

The equations for the second mass and spring are now changed to

$$T_2 = k_2(y-z)$$

$$m_2p^2z = -\mu_2m_2pz+(T_2-T_3),$$

and for the first spring and mass there is no change:

$$T_1 = k_1(x-y)$$

$$m_1p^2y = -\mu_1m_1py+(T_1-T_2)$$

The extension to fig. 20 would therefore involve the provision of an additional second-order loop for w, and additional connections between the loops to allow for the interactions of the different tensions. Any number of loops can be added by repeating this process.

3.3 COUPLED SYSTEM WITH "DASH-POT" FRICTION

Another modification of fig. 15 is shown in fig. 22, in which the friction due to the rough table has been replaced by dash-pot friction. The equations of motion are:

$$
\begin{aligned}
F_1 &= \lambda_1 p(x-y) \\
F_2 &= \lambda_2 p(y-z) \\
T_1 &= k_1(x-y) \\
T_2 &= k_2(y-z) \\
m_1 p^2 y &= F_1 + T_1 - F_2 - T_2 \\
&= T_1 - F_2 - T_2 + \lambda_1 px - \lambda_1 py \\
m_2 p^2 z &= F_2 + T_2 \\
&= T_2 + \lambda_2 py - \lambda_2 pz
\end{aligned}
$$

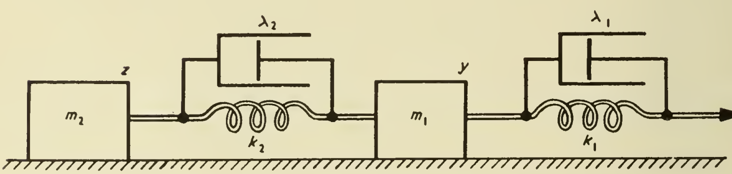


FIG. 22. Coupled System with "Dash-Pot" Friction

A simulator for this system can be built up using the same procedures as before. Assuming initially that a voltage representing y will be available the block diagram for the part of the system comprising m_2, k_2, and λ_2 can be drawn immediately, since it is identical in form with fig. 18 which simulates fig. 12. It is shown in the lower part of fig. 23. For the part of the simulator representing m_1, k_1, and λ_1 the normal procedure would be to assume that there is available, at the output of summing amplifier 2, a voltage proportional to mp^2y, but the above equations show that this would require as one of the inputs a voltage proportional to $\lambda_1 px$, and this would need a differentiator. Assume, therefore, that the output of

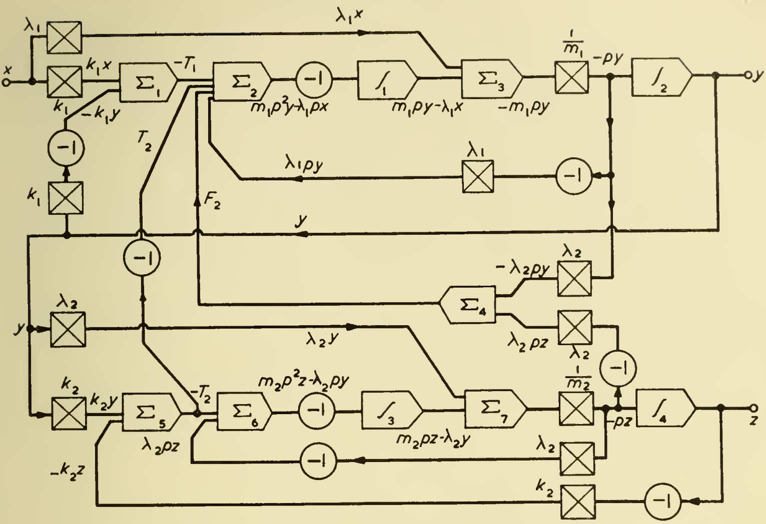


FIG. 23. Simulator for System of Fig. 22

the amplifier represents $m_1p^2y - \lambda_1px$, and remove the unwanted x term as before after the first integrator. The remaining inputs are all available, and may be combined as shown. Since both py and pz appear explicitly it is possible in this case to produce a voltage proportional to F_2, and this is shown in the upper part of the diagram

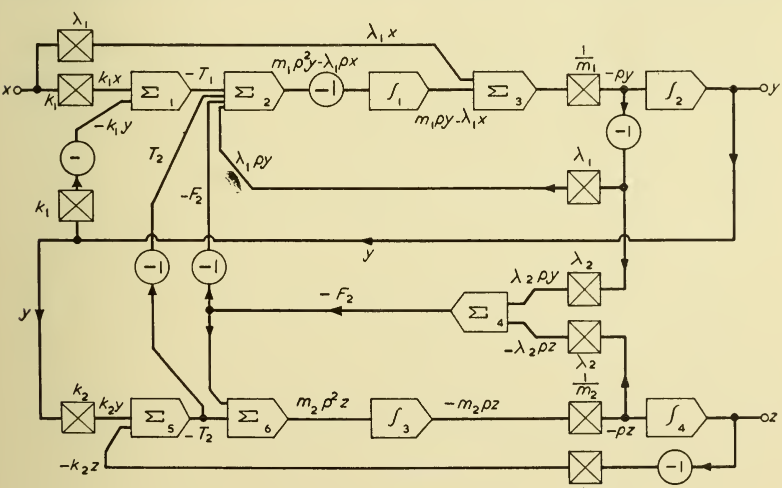


FIG. 24. Modified Form of Fig. 23

as one of the inputs to summing amplifier 2. If desired this voltage can also be used as one of the inputs to amplifier 6 in the lower part of the diagram, as shown in fig. 24. This gives some small economy in computing elements, and it also means that the only points in the simulator where the coefficient λ_2 appears are in the input leads to amplifier 4. Thus a change in the value of λ_2 can be accommodated simply by changing the gain of this amplifier. In a similar way the gains of amplifiers 1 and 5 can be changed to represent changes in the values of k_1 and k_2. Changes in the values of m_1 and m_2 require changes in the multipliers following amplifier 3 and integrator 3 respectively, and in practice this could be done by changing gains or time constants. A change in the value of λ_1 requires changes in two multipliers, one in the "feed forward" of x and the other in the feedback of py.

3.4 ROAD VEHICLE SUSPENSION

The techniques described in the preceding sections, using summing amplifier, reversing amplifier and integrator, can be used to solve practical problems, and some examples will now be given. Although these examples are reasonably plausible it should perhaps be emphasized that the aim is to illustrate the application of analogue computing elements and methods rather than to show how to solve particular problems.

The first problem concerns the suspension system of a road vehicle, (Ref. 6) assumed to be running over a rough road. Fig. 25 shows the parts of the suspension which will be considered here, *viz.* the tyre, the wheel, and the road spring and damper or shock absorber. Considered as a dynamic system, the input disturbances are the variations in the level of the road, measured from some mean position, and these disturbances are transmitted via the tyre to the "unsprung mass" of the axle, linkage, etc., and thence via the road spring and damper to the body of the vehicle. Assuming that the mass of the tyre can be lumped with the unsprung mass the system corresponds to that of fig. 22, where k_1 represents the elasticity of the tyre, λ_1 represents the frictional losses in the tyre, m_1 is the unsprung mass, k_2 and λ_2 represent road spring and damper, and m_2 is an appropriate fraction of the mass of the vehicle. Thus the simulator of fig. 24 can be used for this problem provided steady voltages proportional to m_1g and m_2g (where g is the acceleration due to gravity) are added to the inputs of amplifiers 2 and 6 respectively to represent the weights of the two masses. The output quantity y of this simulator gives the height of the centre of the wheel, measured from a horizontal datum line a distance r_0 above the reference line

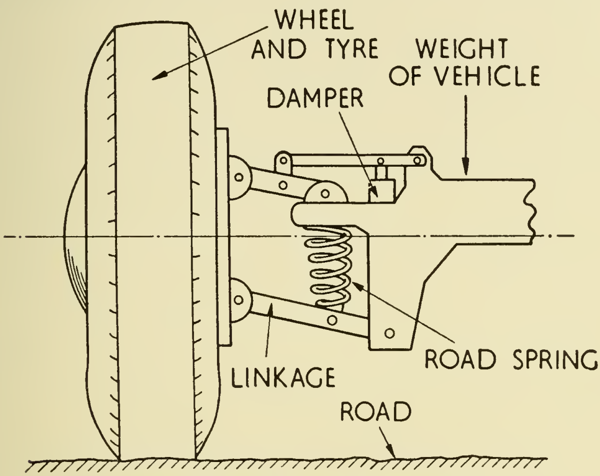


FIG. 25. Road Vehicle Suspension

for x, where r_0 is the outer radius of the tyre in its unstressed condition. Thus whenever $y = x$ the tyre is only just in contact with the road, and exerting zero pressure. The output quantity z is the height of the corner of the vehicle, measured from a datum line displaced from the x datum by an amount defined by the unstressed condition of tyre and road spring.

If x were a step function complete solutions of the problem could be obtained by ordinary analytical methods. However, a vehicle travelling along an ordinary road is not normally subjected to step functions of road level, but to continuous and somewhat random changes which vary in character and magnitude with the "goodness" or "badness" of the road, and in these circumstances the only "paper" method would be a step-by-step numerical solution. The simulator will give the required solution without change, provided a voltage varying with time in a manner representative of the road surface variations is available at the x terminal of fig. 24. As before, the simulator gives a complete picture of the dynamic system, and the behaviour of all the elements of the system can be watched. Different sets of parameters, including variations of load of the vehicle, different proportions of sprung and unsprung mass, different settings of the suspension damper, etc., can be set into the simulator without difficulty. The simulator could be extended to study two wheels, or all four wheels simultaneously, by using two or four sets of amplifiers and integrators as shown in fig. 24, with appropriate interconnections.

It is instructive to imagine that, with the value of x at zero, the effects of gravity can be "switched off". The gravity voltages are removed from the inputs of amplifiers 2 and 6, so that there is no input to the simulator, and all the amplifier and integrator outputs, including y and z, are assumed to be zero. Suppose now that gravity can be restored to the wheel and tyre only. This is simulated by switching the m_1g voltage to the input of amplifier 2, and the resulting output voltage, after two integrations, produces a change of y. Examination of the diagram shows that the value of y will be negative, which means that the wheel will move downwards, as would be expected. The appearance of a y voltage means that voltages representing T_1 will appear at the input of amplifier 1, and this will decrease the net output of amplifier 2. A stage will ultimately be reached where the T_1 voltage exactly neutralizes the m_1g voltage, so that the output of amplifier 2 will be zero, and the value of y will remain constant. This corresponds, of course, to the condition where the tyre has been compressed to an extent sufficient to support the weight of the unsprung mass. During the period while y is changing, frictional forces represented by the F_2 and $\lambda_1 py$ inputs to amplifier 2 will appear. Also, the tension T_2 will change from its initial zero value for a time, and this, with F_2, will cause z to change. In the final state, however, z will assume the same value as y, and both will be constant; then T_2 and F_2 will both be zero. This corresponds with the dynamics, because the vehicle cannot, in the absence of gravity, exert any steady force, though it still has inertia and so can exert inertial forces. If there is no steady force on the spring it must be in its unextended condition, which means that wheel and vehicle must have moved through equal vertical distances.

A similar set of events can be described when the m_2g voltage is switched on to amplifier 6.

This discussion has assumed that the vehicle problem is linear, but in fact, of course, there will be non-linearities; in particular there will be a discontinuity when the tyre bounces out of contact with the road. This effect, and other non-linearities could be built into the simulator by methods which will be described later (Section 4.3).

3.5 MOTION OF AN AEROPLANE

As a further example of the application of a simulator using only the basic amplifier and integrator elements, consider now the equations describing the longitudinal motion of an aeroplane. Assume that the aeroplane is initially flying along a straight line, not necessarily horizontal, at constant speed; but subsequently small movements of the elevator are made, which cause the aeroplane to diverge from the straight line, though it always remains in the vertical plane containing the line. Then the motion can be described by the following three equations (Refs. 7 and 8);

$$\left.\begin{aligned} m(\dot{u}+w_o q) &= -mg\cos\theta_o\cdot\theta+uX_u+wX_w \\ m(\dot{w}-u_o q) &= -mg\sin\theta_o\cdot\theta+uZ_u+wZ_w \\ B\dot{q} &= wM_w+qM_q+\eta M_\eta \end{aligned}\right\} \qquad (19)$$

in which

θ_o is the inclination to the horizontal of the initial flight direction.

θ is a small variation in this angle.

u_o is the initial velocity of the aircraft along the direction of its longitudinal (or "x") axis.

w_o is the initial velocity along the z-axis, which is perpendicular to the longitudinal axis and in the plane of symmetry of the aeroplane.

u and w are small variations in u_o and w_o.

q is the angular velocity about the y-axis, which is perpendicular to the plane of symmetry.

m is the mass of the aeroplane.

B is the moment of inertia about the y-axis.

g is the acceleration due to gravity.

η is the angle of deflection of the elevator.

X_u, X_w are aerodynamic derivatives representing respectively the forces along the x-axis per unit of u and w.

Z_u, Z_w are respectively the forces along the z-axis per unit of u and w.

M_w, M_q, M_η are respectively the moments about the y-axis per unit of w, q, and η.

Fig. 26 shows some of these quantities.

Besides the assumptions mentioned others have been implied and made tacitly. The set of equations given is not intended to represent any particular aircraft, and for other purposes some other equations, including aerodynamic derivatives here neglected, may be more

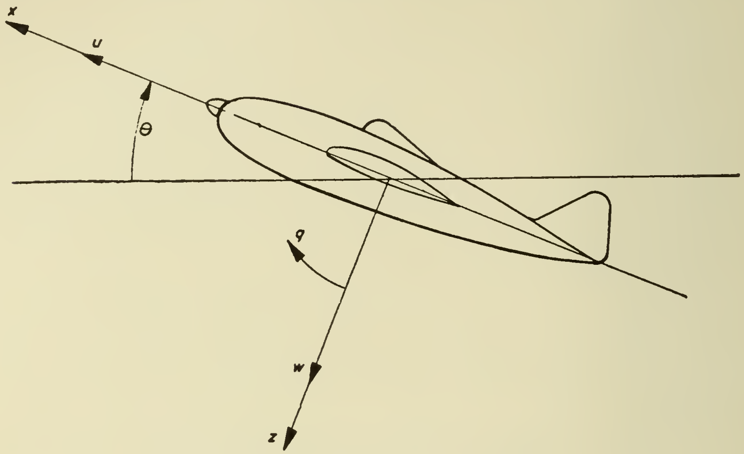


FIG. 26. Motion of an Aeroplane

appropriate. The sign convention follows the standard practice for aerodynamic work.

Setting up the block diagram for the simulator is a straightforward procedure. First, assume that there are three summing amplifiers, one associated with each of the equations (19) giving

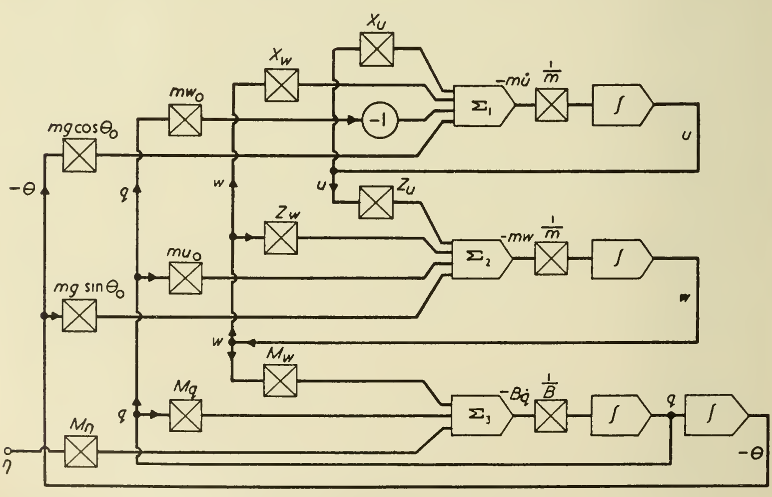


FIG. 27. Aeroplane Simulator

output voltages respectively representing the highest-order derivative in each equation multiplied by its appropriate mass or inertia. These terms are $m\dot{u}$, $m\dot{w}$, and $B\dot{q}$, but it is slightly more convenient to take the negatives of these terms, and this will be done here. The first summing amplifier (fig. 27) is followed by a multiplier to multiply by $1/m$, and the voltage representing $-\dot{u}$ is fed into an integrator of unity time constant to give an output representing u. As before, the required multiplying action would be obtained in practice by adjustment of the integrator time constant. The same arrangement is used with the second summing amplifier, giving an output voltage representing w. For the third amplifier the multiplying factor is $1/B$, and the output of the integrator represents q. From the definition of q and θ it follows that $q = d\theta/dt$, so if the voltage representing q is fed into another integrator the output voltage will represent $-\theta$.

All the variables are now available for making up the input voltages for the summing amplifiers by passing the u, w, q, and $-\theta$ voltages through suitable multipliers. Again, the multipliers would not exist explicitly but would be introduced, in effect, by adjustment of amplifier gains.

It is important to notice that in setting up the diagram of fig. 27 it has been assumed that all derivatives X_u, etc., are positive numbers. In fact, all the derivatives shown except X_w are usually negative numbers, so that the block which calls, for example, for multiplication by X_u implies multiplication by a negative number. Reversal of sign cannot be achieved merely by changing the gain of an amplifier or the time constant of an integrator, so that reversing amplifiers will be required. The most obvious arrangement would be to insert a reversing amplifier at each point where the diagram calls for multiplication by a derivative which has a negative value. However, some economy can be achieved by re-arrangement. As an intermediate step to re-drawing the diagram to take account of the negative signs it is helpful to re-write the equations using the moduli of the derivatives, e.g., for the first of equations (19)

$$m(\dot{u} + w_o q) = -mg \cos \theta_o \cdot \theta - u|X_u| + w|X_w|$$

The arrangement of fig. 27, modified by reversing amplifiers as required, gives a reliable model of the behaviour of the aeroplane in its longitudinal mode, in so far as this is represented by equations (19), and as before, all the variables in the equations appear explicitly as voltages. Changes of parameters can easily be made, and the effects of different values of derivatives, or of mass, or inertia, on the response of the aeroplane to deflections of the elevator, can be studied.

Among the assumptions implied in the equations (19) is that of complete linearity; for example, the constancy of the derivative X_u implies that the force uX_u varies linearly with u. In fact, the variation is not usually linear and means for introducing non-linear effects will be described later.

The arrangement of fig. 27 can be used as a basis for the study of some problems in the automatic control of aeroplanes. For example, suppose that in a blind-landing system the preferred line of flight is a straight line inclined at an angle θ_0 to the horizontal, θ_0 being in this case small and negative. Then this line may be regarded as the line of flight used in describing equations (19), and an aircraft flying with constant speed along this line satisfies the initial conditions associated with these equations. Suppose that if the aeroplane diverges from the line, but remains in the same vertical plane, the radio equipment in the aeroplane gives out a voltage which is, say, of positive or negative sign depending on whether the aeroplane is above or below the line, and which has a magnitude proportional to the displacement. This voltage is used to move the elevators in such a way that the displacement tends to diminish. There are various ways in which this can be done; the voltage may be used to give some indication to the pilot, who then moves his control column in the appropriate manner; or in a completely automatic system the voltage may be fed to an autopilot which in turn operates the elevator. For the present purpose the pilot, whether human or automatic, will be ignored, and the elevator deflection η will be taken to be proportional to the voltage, giving

$$\eta = kh$$

where h is the displacement of the aeroplane from the line, and k is a constant.

Now from equations (19) it can be shown that the acceleration of the aeroplane along the z axis is equal to

$$\ddot{z} = \dot{w} - u_0 q$$

Assuming that θ remains small and that the x axis of the aeroplane remains approximately parallel to the direction of flight,

$$\ddot{h} = \ddot{z} = \dot{w} - u_0 q \text{ very nearly.}$$

Hence

$$\eta = k\int\int(\dot{w} - u_0 q)\,dt\,.\,dt$$

Now voltages representing both $-\dot{w}$ and q are available in the simulator of fig. 27, so to extend this simulator to study the automatic blind-landing system it is only necessary to take the sum of $-\dot{w}$ and q in a summing amplifier integrate twice, and feed the output, which now represents h, into the η terminal of fig. 27. The complete arrangement is shown in fig. 28. In this diagram account

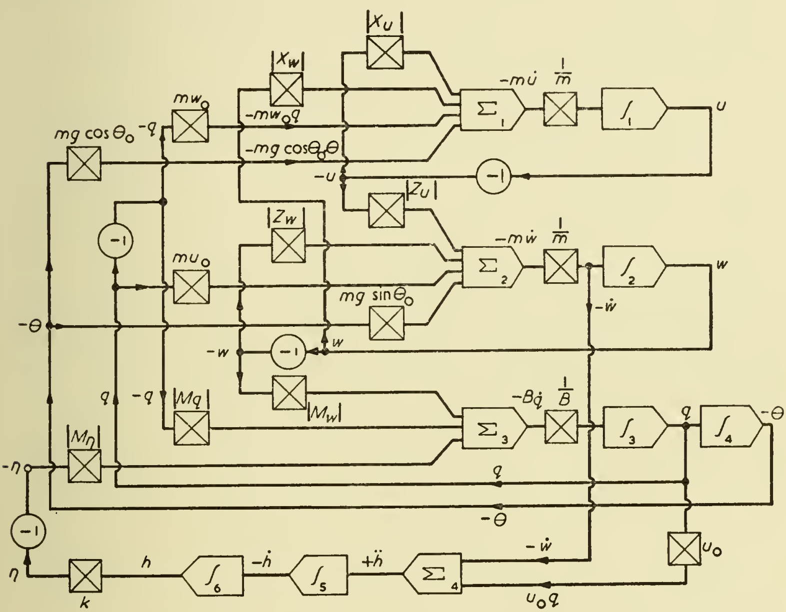


FIG. 28. Simulator for Aeroplane with Auto Control

has been taken of the signs of the aerodynamic derivatives, in accordance with an earlier paragraph, and the derivative multipliers are now labelled $|X_u|$ (=modulus of X_u), etc. By re-arranging the connections the negative signs can be accommodated with three additional sign-reversing amplifiers.

The simulator of fig. 28 now represents, in the language of servomechanisms, a "regulator", which is intended to cause the aeroplane to fly along a fixed straight line, and there is no input terminal for a signal from an external source as there was in fig. 27. In making a study of a system of this kind the interest lies in observing first of all whether the system is stable. With the arrangement shown the system would almost certainly be unstable, but in practice a more complex relation between h and η would be used to ensure stability. With a stable system interest lies in observing whether the aeroplane flies as close as desired to the fixed line, in the presence of whatever disturbing forces might occur. The most obvious kind of disturbance is a wind gust, and the effect of this can be studied, provided the variation of wind speed due to the gust can be adequately described in terms of the quantities available in the simulator. To take a very simple case, suppose that the gust changes the wind speed for a short period from zero to some fixed value and then back to zero, the change occurring in the x direction only, so that the only immediate effect is to change the value of u. This can be simulated by replacing the sign-reversing amplifier operating on the u voltage by a summing amplifier and arranging this to add to the output of integrator 1 a voltage representing the gust velocity. The voltage is controlled by a switch, and if the switch is closed for a period equal to the duration of the gust the appropriate change will be made in the value of u, and the disturbance will be injected into the system. It is assumed throughout this discussion that the wind gust is of short duration; interest is confined to the disturbance of the lateral motion, and the effect on the ground speed, for example, is ignored.

The assumption that the gust has an effect in the x direction only is somewhat unreal, since it implies that the gust direction is inclined to the horizontal. A more realistic gust would change the wind velocity along a direction parallel to the ground, and to take account of this in the simulator involves resolution of the wind velocity into two components in the x and z directions. Provided the two components are known the disturbance in u can be injected as before, and the disturbance in the z direction can be injected by a corresponding modification to the w voltage. However, the resolution, if accurately performed, requires multiplication by $\cos(\theta_0 - \theta)$ and $\sin(\theta_0 - \theta)$, and these quantities change continuously with the pitching motion of the aeroplane. This type of resolution cannot be achieved with the simple computing elements so far described, although means are available and will be described later.

If it is sufficiently accurate to take a mean value for $(\theta_0 - \theta)$, i.e. θ_0, then the effect of the horizontal gust can be simulated. The voltage to be added to the output of integrator 1 is now proportional to the gust velocity multiplied by cos θ_0, and a voltage proportional to the gust multiplied by sin θ_0, is to be added to the output voltage

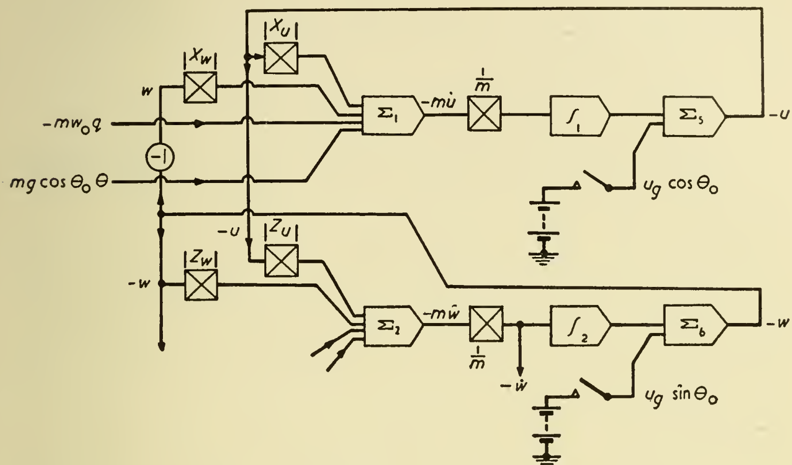


FIG. 29

of integrator 2. Fig. 29 shows the relevant parts of fig. 28, with the additions necessary to introduce the gust, whose velocity is taken to be u_g. Some small changes in the arrangement of the sign-reversing amplifiers are necessary to preserve correct signs for the w and $-w$ voltages.

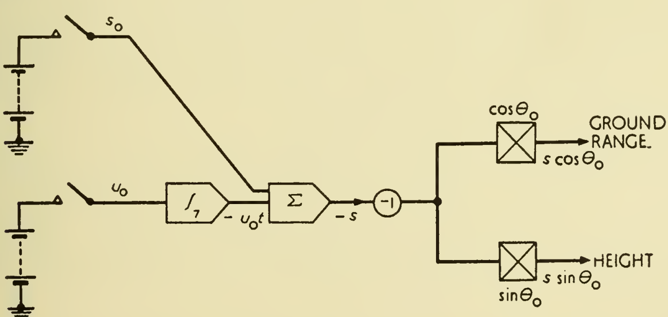


FIG. 30

The simulation of the blind-landing problem can be made a little more complete by a representation of the motion of the aeroplane along the inclined line. For the undisturbed motion the immediate practical value of this is trivial but as is often the case in building up simulators it is worth considering as a basis which can be modified to represent more complex conditions. The required equipment is shown in fig. 30, and it is mainly an integrator and summing amplifier to solve the equation

$$s = s_0 - u_0 t = s_0 - \int_0^t u_0 \, . \, dt$$

where s is the range of the aeroplane along the inclined line, measured from the point where this line meets the ground, and s_0 is the initial value of s, at $t = 0$.

The output of the summing amplifier represents $-s$, and after sign reversal it is split into two channels, one of which is multiplied by $\cos \theta_0$ to give the ground range of the aeroplane while the other is multiplied by $\sin \theta_0$ to give the height. Since $\cos \theta_0$ and $\sin \theta_0$ are both less than unity the multipliers for these two quantities may be ordinary potentiometers.

If it is required to take into account in fig. 30 the effect of a gust, then the most obvious method is to add a voltage representing $u_g \cos \theta_0$ to the input of integrator 7. This will then give a correct value for the slant range s (except for the error involved in assuming that the x-axis of the aeroplane remains parallel to the inclined line), and for some purposes the value of ground range given by $(s \cos \theta_0)$ will still be sufficiently accurate if θ_0 is a fairly small angle. The height as given by $s \sin \theta_0$ will, however, be in error by an amount $h \cos \theta_0$, where h is the distance of the aeroplane from the inclined line, measured perpendicular to the line. The correct height could be obtained by taking a voltage from fig. 28 representing h, multiplying by $\cos \theta_0$ by means of a potentiometer, and combining with the voltage representing $s \sin \theta_0$ in fig. 30.

Alternatively, the arrangement of fig. 30 may be replaced by two similar arrangements, one to compute the ground range and the other to compute the height. The voltage source representing u_0 is replaced by two sources, one representing $u_0 \cos \theta_0$ and the other $u_0 \sin \theta_0$, and the final multiplications by $\cos \theta_0$ and $\sin \theta_0$ are not required.

With these last modifications the simulator has now grown to a moderate size, using about twenty amplifiers and integrators. In practice, if it were required to extend the simulation still further this would probably be a suitable point at which to review the whole of the real problem, and the simulation of the problem. Such a review would have two main objectives. The building up of a complex simulator by a series of additions to a simple basic simulator is a convenient and straightforward procedure, and it has the further advantage that the size of the simulator can grow and the accuracy of representation of the real problem can be improved as the operator's knowledge and appreciation of the problem grow. However, it may well happen that this piecemeal development may not give the best arrangement for a simulator of a given degree of complexity. There is some suggestion of such a situation developing in the blind-landing simulator just discussed, in which any further extension might require that the two sets of axes involved—one set fixed to the aeroplane and the other set fixed to the ground—should be represented fully, instead of only partially as in figs. 28 and 30. If this were so it would be desirable to re-arrange the simulator on this basis rather than think only of extensions to an existing arrangement.

A second reason for a review is to avoid undue accumulation of errors. In any kind of computation in which the complexity of the problem being solved tends to increase from some relatively simple starting point it is important to ensure that any approximations made in the earlier stages remain valid. This is particularly the case in building up a simulator in the manner described above, since the comparative ease with which extensions can be made make it easy to overlook any approximations built into the simulator at some earlier stage. By contrast, the additional labour consequent on raising the order of differential equation to be solved on paper is often so great that a very careful examination would be made to ensure that the improvement in the accuracy of representation of the problem would be sufficient to justify the extra labour.

As an example of the way in which errors can be introduced by extending the simulator, consider the extension of fig. 30 to give height and ground range in the presence of a gust. At first sight, since the only immediate effect of the gust is to change velocities and since furthermore the only velocity appearing in fig. 30 is u_o, it might be thought that the only requirement would be to modify u_o in an appropriate manner. However, as seen above, although this gives a reasonable value for the ground range, provided θ_o is small, it may give only a poor approximation for height unless account is also taken of the displacement of the aeroplane from the inclined line.

4

SIMULATORS FOR NON-LINEAR PROBLEMS

The examples of applications of simulators given so far have all been made up from the three basic elements, *viz.* summing and reversing amplifiers and integrators, and they have all applied to problems which can be represented by linear differential equations with constant coefficients. These examples have been of value in demonstrating some features of simulators of this type, but since all the equations could, at least in principle, be solved by ordinary analytical methods, the advantages would probably not be great enough to justify general adoption of simulator methods for the solution of such problems. In certain cases it might be worth while to use a simulator if it were available, or even to build one specially, to solve problems having an arbitrary input, such as the road vehicle suspension problem, or problems involving equations of high order, such as the blind-landing problem, for which a large number of solutions were needed.

However, the usefulness of a simulator, compared with paper methods, is greatly enhanced when problems are to be solved which cannot be described in terms of linear differential equations with constant coefficients. For this purpose additional computing elements are required, and some of these will now be mentioned. Fuller description of the operation of these computing elements will be given later (Chapter 7); for the present purpose a brief outline of their capabilities will suffice.

The multiplier is a device which has two input terminals into which are fed two independent voltages. The output voltage is proportional to the product of the two input voltages, whether they are constant or variable with time. This kind of multiplier will be called a "two-variable" multiplier, when necessary, to distinguish it from the "multipliers" which have already been used for multiplying by constants.

By feeding the same input into both input terminals of a two-variable multiplier the square of a quantity can be obtained. There is a related device which will give the square root of an input quantity, and another which will divide one input quantity by a second input quantity, giving an output voltage proportional to the quotient.

The sine computer gives an output voltage proportional to $\sin \theta$ when fed with an input voltage proportional to θ. The cosine computer correspondingly gives an output proportional to $\cos \theta$. Related to these computers are the sine and cosine potentiometers, which give outputs proportional respectively to $\sin \theta$ and $\cos \theta$ when the angular position of a shaft is set to represent θ.

The arc-tan computer gives an output voltage proportional to the angle whose tangent is the ratio of two input voltages, i.e. if the two input voltages are V_x and V_y, the output voltage is proportional to arc-tan (V_y/V_x).

The function-generator, or curve-follower is fed with a voltage proportional to x and gives as an output a voltage proportional to some function $f(x)$. The particular function required must, of course, be set into the generator before use, but with appropriate changes and adjustments most types of generator can be used for a variety of different functions. Usually only single-valued functions can be handled, but with this restriction almost any continuous function can be reproduced, including trigonometric functions, parabolas for squaring variables, and also curves based on experimental results.

The limiter is a device which gives an output voltage equal to the input voltage when the input is below some predetermined level, but remains fixed at this level whenever the input exceeds this level; i.e. if V_1, V_O, V_L, are respectively the input, output and limiting voltages,

$$V_O = V_1, \quad V_1 < V_L$$
$$V_O = V_L, \quad V_1 \geqslant V_L$$

The "trigger" is an electronic device which has two quiescent states, and transition from one state to the other occurs very rapidly if an input voltage passes through some critical value. Transition from one state to the other causes a re-distribution of currents in the system, so that, for example, a relay can be caused to operate at the instant of transition.

4.1 A BALLISTICS PROBLEM

Some examples will now be given of simulators which need to use one or more of these elements to solve non-linear or "variable-coefficient" problems.

Take first a simple problem in ballistics, in which a gun, elevated at an angle θ to the horizontal, fires a missile with muzzle velocity V_M. The mathematical treatment of the problem *in vacuo* is simple. The vertical component of the initial velocity is $V_{VO} = V_M \sin\theta$, and the vertical component of the instantaneous velocity at any time t after the firing of the gun is equal to

$$V_V = V_{VO} - gt$$

where g is the acceleration due to gravity.

The instantaneous height h is given by

$$h = \int_0^t V_V . dt = \int_0^t (V_{VO} - gt)\, dt$$

or,

$$h = V_{VO}t - \tfrac{1}{2}gt^2$$

The initial value of the horizontal component of velocity is $V_{HO} = V_M \cos\theta$, and the instantaneous value V_H at any time after firing is equal to this initial value, since there is no horizontal force to provide deceleration. The horizontal range at time t is therefore

$$s = \int_0^t (V_{HO})\, dt = V_{HO}t$$

A simulator for the solution of this simple ballistics problem is, of course, unnecessary, but as a basis for extension to more difficult cases a suitable arrangement is shown in fig. 31. The two parameters which might require to be changed are the muzzle velocity V_M, which is represented by an adjustable battery voltage, and the angle of elevation θ. In fig. 31 the multiplication by $\sin\theta$ and $\cos\theta$ could be achieved, as before, by means of two ordinary potentiometers; but a more convenient arrangement is to use a sine potentiometer and a cosine potentiometer, and to mount these on one shaft so that a single adjustment for θ sets $\sin\theta$ and $\cos\theta$ simultaneously.

To find the range at which the missile strikes the ground, which is assumed to be horizontal, it is necessary to find the value of s at the instant when h passes through zero. There are two general methods by which this may be done. First, the two voltages representing h and s are recorded simultaneously by means of a moving-pen recorder or an oscillograph or by a cathode-ray oscilloscope fitted with a camera, with arrangements to provide on the records suitable scales of amplitudes and time. The "flight" of the missile is allowed to continue until it is certain that h has passed through zero. Then on examination of the records it will be possible to observe the value of s when h passes through zero. The second method depends on the

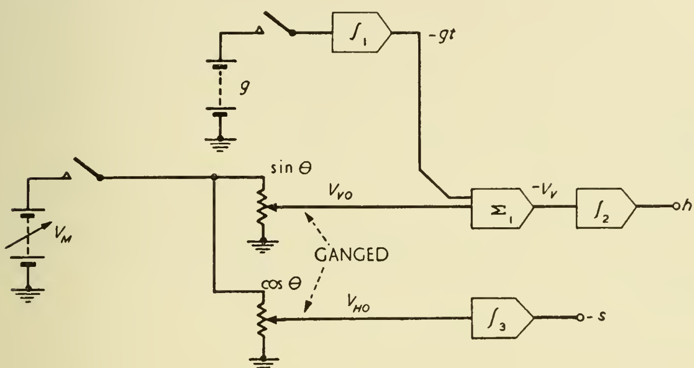


FIG. 31. Simulator for Ballistics Problem

use of the trigger device to operate a relay when the input voltage passes through some critical value. For the present purpose the input voltage is the h voltage, and the critical value is zero. The relay is used to reduce to zero the input voltage to integrator 3 in fig. 31, so that the output voltage remains at the value corresponding to $h = 0$. Although the trigger and relay operate very rapidly, the operating time may still be long enough to cause an appreciable error. This can be reduced by arranging the "critical value" of input voltage to be such that operation of the trigger and relay begins just before h reaches zero and the relay contacts close as h passes through zero. The exact value of "anticipation" can be determined either by measurement of the trigger delay, or by simulating a ballistic problem whose answer is accurately known.

The most serious disadvantage of this treatment of the ballistics problem is that it ignores air resistance, and the simulator can be extended to take account of this. For a given missile the resistance of the air depends on the velocity of the missile. The relationship is not simple, but over a certain range of conditions the force of resistance F may be taken as proportional to the square of the velocity, i.e.

$$F = RV^2$$

where R is a constant and V is the instantaneous velocity of the missile. In the presence of air resistance the previous assumption of constant horizontal component of velocity will not hold, and it will now be a variable, V_H.

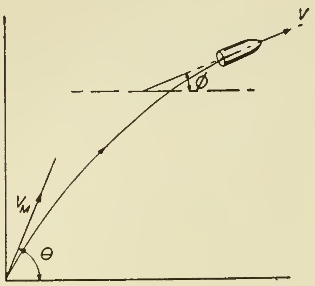


Fig. 32

The resistance F can be resolved into vertical and horizontal components, and the vertical component of acceleration of the missile, taking the air resistance into account is

$$-(g+F\sin\phi)$$

where ϕ is the instantaneous direction of flight of the missile relative to the horizontal (fig. 32). The horizontal component of acceleration is

$$-F\cos\phi$$

Also

$$V^2 = V_V^2 + V_H^2$$

$$\tan\phi = \frac{V_V}{V_H}$$

At any time t after the firing of the gun the horizontal component of velocity of the missile is

$$V_H = V_{HO} - \int_0^t F\cos\phi\,dt$$

and the vertical component is

$$V_V = V_{VO} - \int_0^t (g+F\sin\phi)\,dt$$

The simulator is shown in fig. 33. Assuming at first that voltages representing $F\sin\phi$ and $F\cos\phi$ are available the arrangement of fig. 31 is modified by adding $F\sin\phi$ to the input voltage of integrator 1 to give the new vertical acceleration, and an additional amplifier (2) and integrator (4) are needed to take account of the horizontal component of resistance $F\cos\phi$. The output voltages of amplifiers 1 and 2 now give the new values of V_V and V_H, and integrators 2 and 3 give the height h and ground range s as before. The next step is to compute V^2 and this is achieved by squaring V_V and V_H and adding. To square V_V the voltage representing V_V

is fed into both input terminals of a two-variable multiplier, giving an output V_V^2. Another multiplier gives V_H^2, and the addition to give V^2 takes place in amplifier 3. Multiplication by R, which would

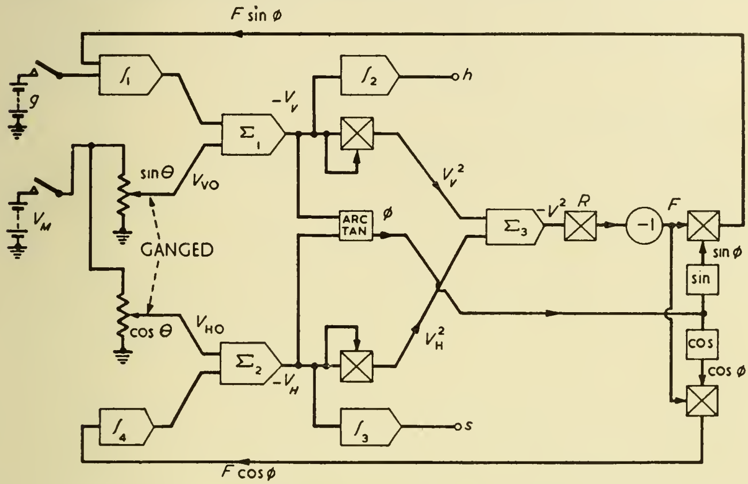


FIG. 33. Simulator for Ballistics Problem with Air Resistance

in practice be achieved by adjustment of the gain of amplifier 3 coupled with a suitable choice of scale factors, gives $RV^2 = F$. Further taps from the outputs of amplifiers 1 and 2 are fed into an arc-tan computer giving an output $\phi = \text{arc-tan } (V_V/V_H)$. This voltage is fed into sine and cosine computers giving $\sin \phi$ and $\cos \phi$, and two more two-variable multipliers give $F \sin \phi$ and $F \cos \phi$.

The arrangement of fig. 33 can be further extended to deal with more complex versions of the ballistics problem. If the resistance function is of some other form than that assumed, appropriate changes can be made. If the function contains odd powers of V then a square-root computer is needed. If the function is such that it cannot be represented as the sum of powers of V, then a curve follower is required, to give an output proportional to F when fed with a voltage proportional to V^2 or to V. The effect of change of air density with height can be included by replacing the resistance equation $F = RV^2$ by a more complex equation giving F as a function of V and h. If the function is fairly simple the simulation can probably be achieved by using additional summing amplifiers and multipliers, but a curve-follower may be necessary for complex functions.

The effect of a horizontal wind blowing directly up-range or down-range, i.e. in the plane containing the vector V, can easily be taken into account by adding or subtracting a voltage proportional to the wind velocity, V_W, to the voltage representing V_H which is fed into multiplier 2 in fig. 33. The effect of this is to replace the resistance equation

$$F = RV^2 = R(V_V^2 + V_H^2)$$

by another equation

$$F = R\{V_V^2 + (V_H \pm V_W)^2\}$$

With this arrangement the wind velocity need not be constant; the effects of varying velocity can be reproduced by varying the "wind voltage" in an appropriate manner. If the wind direction is not in the plane of V the problem becomes three-dimensional instead of two-dimensional and the simulation becomes more complex. However, no new principles are involved, and the simulator block diagram can be drawn without difficulty if the appropriate set of simultaneous equations is first written down. At the expense of further complications, but again without introducing any new principle, the gyroscopic effects of a spinning shell, and the effects of curvature and rotation of the earth could be taken into account.

4.2 MOTION OF AN ELECTRON

As an example of a dynamic system in a different field of science, consider the motion of an electrically-charged particle in electric and magnetic fields. Suppose that gravity may be neglected, and that the particle moves in an otherwise completely empty space at speeds small enough for relativistic effects to be ignored. Assume at first that the fields are uniform, with the magnetic field H parallel to the z-axis in a Cartesian co-ordinate system, and the electric field F in a direction defined by the angles θ and ϕ in Fig. 34. If the mass of the particle is m, its charge q, and its velocity v, then its acceleration f is given (Ref. 9) by the vector equation:

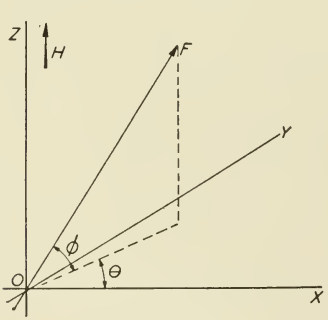


FIG. 34.
Electric and Magnetic Fields

$$\frac{m}{q} f = F + v \times H$$

For simulator solution this is split into three separate equations:

$$\frac{m}{q}\ddot{x} = F_x + H_z\dot{y} - H_y\dot{z}$$

$$\frac{m}{q}\ddot{y} = F_y + H_x\dot{z} - H_z\dot{x}$$

$$\frac{m}{q}\ddot{z} = F_z + H_y\dot{x} - H_x\dot{y}$$

where F_x, etc., are the components of F in the directions of the axes. Fig. 34 shows that

$$F_x = F\cos\phi\cos\theta$$

$$F_y = F\cos\phi\sin\theta$$

$$F_z = F\sin\phi$$

The choice for the direction of H gives

$$H_x = H_y = 0$$

The simulator consists of three main sections, one associated with each co-ordinate, and one section is illustrated in fig. 35. This section computes $\ddot{y}$ and $\dot{y}$ from F_y and $H_z\dot{x}$. The $\dot{x}$ term is provided by another section of the simulator, not shown, and the $\dot{y}$ voltage

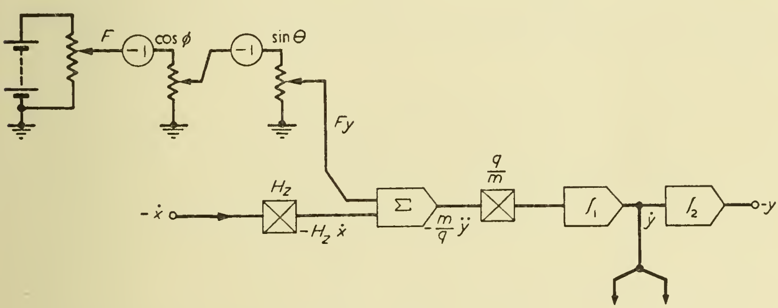


FIG. 35. Section of Simulator for Motion of a Charged Particle

given by the section shown is used in a similar way by the other two sections. The complete simulator needs two sine potentiometers and two cosine potentiometers, and if these are mounted in two "ganged" pairs one shaft rotation represents θ and the other ϕ. The reversing amplifiers preceding the cos ϕ and sin θ potentiometers are inserted to remove "loading" effects, not primarily for sign changing.

The simulator, even as it stands, is not restricted to steady fields; variations of the strength of H or F can be accommodated by the obvious adjustments if the changes are slow. If the changes are rapid, then voltages varying in a proportional manner are required, and these are used to replace the battery shown for F and to provide the second input for a two-variable multiplier to give $H_z\dot{x}$. Slow variations of the direction of F can be accommodated by adjustment of the θ and ϕ shafts, but for more rapid changes proportionally-varying voltages are needed, and the sine and cosine potentiometers are replaced by sine and cosine computers of the type already mentioned. For changes of direction of H it is necessary to introduce the H_x and H_y terms and to provide a set of sine and cosine computers and follow the same procedure as for F.

Non-uniform fields can also be represented. Suppose, for example, that the field H is parallel to OZ, but varies in strength according to some known function of the distance from OZ, i.e.

$$H_z = H_0 f(r), \text{ where } r = \sqrt{x^2+y^2}$$

Then r can be computed by means of two squaring multipliers, a summing amplifier, and a square-root device; or possibly by an arc-tan computer to give $\alpha = \tan^{-1}(y/x)$ and a sine computer and divider to give $r = y/\sin \alpha$ if α remains within a limited range well away from zero. The voltage representing r is fed to a function generator to give $f(r)$ which in turn gives H_z.

4.3 ROAD VEHICLE SUSPENSION WITH "BOUNCE"

An interesting application of the trigger circuit is in the extension of the simulator of fig. 24 (section 3.4) to include the effects of the tyre bouncing out of contact with the road. During the bounce the pressure between tyre and road is zero, and the behaviour of the suspension system will be entirely unchanged if the section of the road traversed during the bounce is replaced by a different section, so shaped that the tyre remains just in contact with the road, but exerting zero pressure, during the whole bounce. At the end of the bounce the tyre will resume contact with the true road surface. The section of road to be used in place of the real road during bounce must have a shape such that it is always distant r_0 below the centre of the wheel, where r_0 is as before the outer radius of the tyre in the unstressed condition. To produce the desired effect in the simulator x is subtracted from y as shown in fig. 36 and the difference voltage is fed into a trigger circuit set to operate when the input voltage passes through zero. If now y increases relative to x so that $(y-x)$ passes through zero the trigger will operate and this will occur when the tyre leaves the road. The trigger operates a relay which switches the x input terminal from the normal source of x voltage, representing

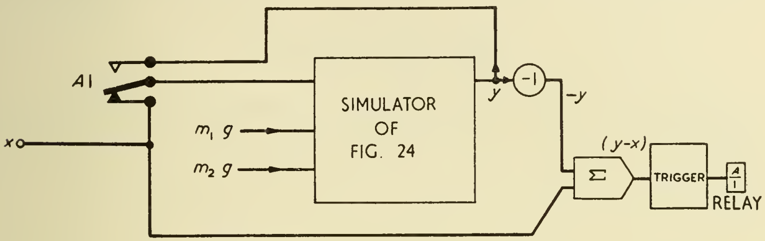


Fig. 36. Simulator for a Road Vehicle Suspension with Bounce

the real road surface, and connects it to the output of an amplifier giving a voltage representing y. Thus the "road" remains temporarily at distance r_0 from below the wheel centre. When $(y-x)$ falls below zero the trigger releases the relay and the x input terminal is reconnected to the normal source of road-surface voltage.

If the simulator were such that an explicit voltage appeared representing the pressure between the tyre and the road it would be sufficient to use the trigger circuit and relay to earth the point in the simulator where this voltage appeared, in accordance with the obvious fact that the pressure is zero when the tyre is out of contact with the road. In the simulator shown in fig. 24 there is no such voltage, so the device of the fictitious road during bounce is used. A voltage representing the pressure between tyre and road could be produced if differentiators were allowed, or if the frictional losses in the tyre were so small that the friction coefficient λ_1 could be neglected. In the latter case it would only be necessary to earth the output terminal of amplifier 1 by means of the trigger-operated relay.

4.4 A NAVAL GUNNERY PROBLEM

As a final example of the application of simulators consider the rudimentary naval "battle" represented in fig. 37. Two ships S and T are sailing with speeds V_S and V_T in directions defined by the angles ψ_S and ψ_T, and the line joining them is of length R and has a direction defined by the angle β. Let the initial co-ordinates of S

and T be (0, 0) and (x_{T_0}, y_{T_0}) respectively. Then the position of S at time t is given by

$$x_S = \int_0^t V_S \cos \psi_S \, dt$$

$$y_S = \int_0^t V_S \sin \psi_S \, dt$$

and these expressions hold whether V_S and ψ_S are constant or variable.

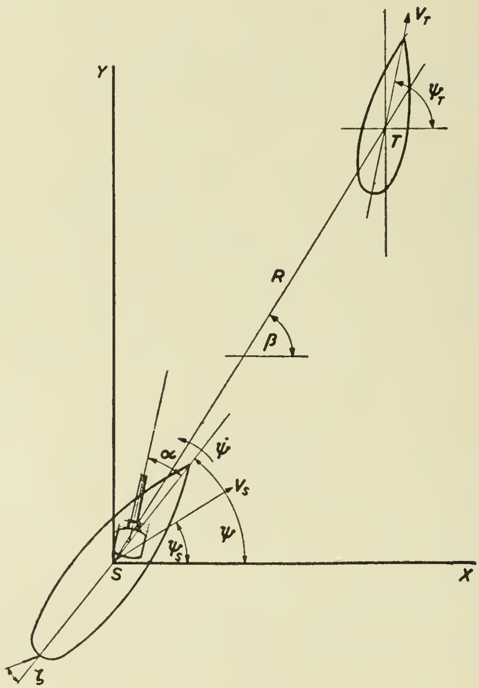


FIG. 37. Naval "Battle"

The position of T at the same instant is given by

$$x_T = x_{T_0} + \int_0^t V_T \cos \psi_T \, dt$$

$$y_T = y_{T_0} + \int_0^t V_T \sin \psi_T \, dt$$

A simple simulator for computing the two positions can be arranged following the procedures already described. The range R can be computed by simulating the equation

$$R = \sqrt{(y_T - y_S)^2 + (x_T - x_S)^2}$$

and β can be found from

$$\beta = \tan^{-1}\left(\frac{y_T - y_S}{x_T - x_S}\right)$$

Suppose now that S is pursuing T, and intends to fire a shell. If S is to sail in the direction of T, S must be turned so that $\psi_S = \beta$, i.e. so that $\beta - \psi_S = 0$. However, when the ship is turning the angle ψ_S which defines the direction in which the centre of gravity of the ship is moving is not equal to the angle ψ which defines the direction of the fore-and-aft line of the ship. There is no direct indication of the value of ψ_S, but the angle $\beta - \psi$ can be found by observing the direction of the target ship T relative to the fore-and-aft line of S, and for the present purpose it will be assumed that the angular deflection of the rudder ζ is made proportional to the angle $(\beta - \psi)$. This will tend to make $(\beta - \psi)$ equal to zero, and this condition means that the fore-and-aft line of S is pointing at T, which is a reasonable direction for the pursuit of T. In practice there would usually be a helmsman forming a link between the observation of the angle $(\beta - \psi)$ and the movement of the rudder, but the reaction time of the man is short compared with the time scale of events in this battle and it will be ignored. The man could be represented, if desired, by inserting a simple lag of about 0·3 second time constant in or after the summing amplifier which adds β and $-\psi$. Alternatively a more complex network might be used; or if the man's behaviour was considered sufficiently important a real man could be included. In the latter case the man would need a visual indication of the angle $(\beta - \psi)$, and he would be provided with a "wheel" which would drive a potentiometer to give the ζ voltage to be fed back into the simulator.

In the steady state $\dot{\psi}_S$ and $\dot{\psi}$ will be equal, but it will be assumed that there is a time lag T_1 between a movement of the rudder of ship S and the corresponding rate of yaw $\dot{\psi}$, and another time lag T_2 between the development of this rate of yaw and the corresponding rate of turn $\dot{\psi}_S$ of the ship's track. Then

$$\left.\begin{aligned} \zeta &= k_1(\beta - \psi) \\ \dot{\psi} + T_1\ddot{\psi} &= k_2\zeta \\ \dot{\psi}_S + T_2\ddot{\psi}_S &= \dot{\psi} \end{aligned}\right\} \qquad (20)$$

Now suppose that the ship S fires a shell at T. At the instant of firing the direction of the gun barrel is at an angle α relative to the fore-and-aft line of the ship, so that the direction of the barrel relative to OX is $\psi + \alpha$. If, at the instant of firing, the ship is yawing at an angular rate $\dot{\psi}$, the end of the gun barrel will have a velocity relative to the C.G. of the ship, and in a direction at right angles to the direction of the barrel. The magnitude of this velocity will be equal to $B'\dot{\psi}$, where B' is the effective length of the barrel, i.e. the "horizontal component" of the length, which is $B \cos \theta$, where B is the true length and θ the angle of elevation. It is assumed here that the gun is attached rigidly to the ship at a point vertically above the centre of gravity, that the ship remains on an even keel, and the yaw takes place about a vertical axis through the centre of gravity. These are all somewhat unrealistic assumptions, but they are convenient for the present purpose, and they could be replaced by sounder assumptions later on. Some effects due to the ship's own speed, etc., have been ignored, but these too could be included if desired.

If the muzzle velocity of the shell is V_M, then on emerging from the muzzle when the ship is yawing, the total velocity of the shell will be the resultant of V_M and $B'\dot{\psi}$, and although the magnitude of the resultant will be substantially equal to V_M, since the other term is small, the direction of the shell will be turned through a small angle $\tan^{-1}(B'\dot{\psi}/V_M) \doteq B'\dot{\psi}/V_M = (B \cos \theta)(\dot{\psi}/V_M)$. Hence the direction of the shell relative to OX is $\eta = \psi + \alpha + (B \cos \theta)(\dot{\psi}/V_M)$.

To study the flight of the shell, a simulator for the two-ship problem, as outlined above, must be combined with the "ballistic" simulator of fig. 33. The block diagram is shown in fig. 38; it follows the same principles as those described earlier, and will not be treated in detail. The upper section represents the motion of the target ship T, with potentiometers for setting the speed, V_T, the initial position (x_{T_0}, y_{T_0}), and the direction of the ship's track ψ_T. V_T and ψ_T can be changed during the problem, if desired. The reversing amplifier following the V_T potentiometer is not inserted for sign-changing, but to remove the "loading" effects which the sine and cosine potentiometers would otherwise impose on the V_T potentiometer.

The next section represents the motion of the ship S, and bears some resemblance to the first section, except that instead of a ganged pair of sine and cosine potentiometers for setting in ψ_T by hand, sine and cosine computers are used to produce voltages representing

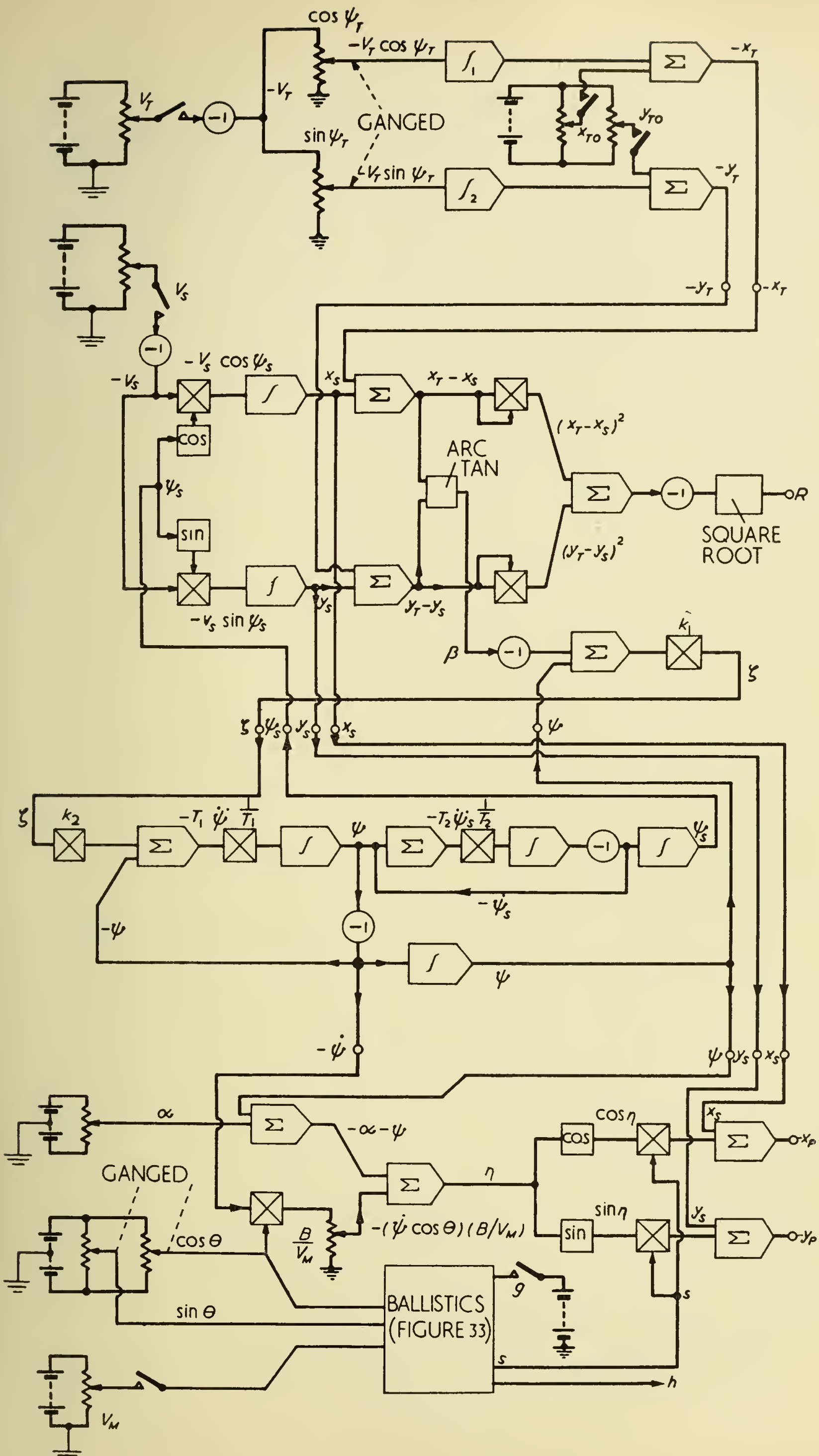


Fig. 38. Simulator for Naval Battle

sin ψ_S and cos ψ_S. This is necessary because ψ_S is not available as an independent variable but is computed from the rudder deflection by the third section of the simulator, which solves equations (20). The remaining section of the simulator computes the angle η, at which the shell flies, and uses this to compute the position of the shell. The block labelled "ballistics" represents the simulator of fig. 33, and the output s is multiplied by cos η and sin η to give the co-ordinates of the position of the shell relative to the ship S. Addition of the co-ordinates of S, which are (x_S, y_S), gives the position (x_P, y_P) of the shell referred to OX and OY, and assuming that either a recorder or a trigger is used to determine when h is zero, the position of the "splash" of the shell relative to T can be found. If a recorder is used, then the position of T must be recorded, and if a trigger is used two sets of relay contacts are needed, one set to "freeze" the position of T by reducing to zero the input voltages to integrators 1 and 2, and the other to "freeze" the value of s by reducing to zero the input voltage of the integrator in the "ballistics" block which gives an output voltage representing s. This integrator corresponds to integrator 3 in fig. 33.

The ships can be set "sailing" by closing the switches controlling V_T, V_S, x_{T_0}, y_{T_0}, and the shell is "fired" by closing the switches controlling V_M and g. In practice other switches or relay contacts would be used to hold the input voltages to integrators and other elements at zero until the computations actually begin. This point will be discussed more fully in Section 8.2.

Although the simulator now contains more than 60 computing elements, further extension is possible. A more realistic means for setting the bearing and elevation of the gun is obviously desirable, and the ballistic computation could be extended to include the effects of wind, variation of air density with height, or different resistance functions. The ship motion could be modified to include the effects of rolling and pitching of the ship, although the problem would then become three-dimensional, and the complexity of the simulator would increase considerably. Information would be required about the rolling, pitching and yawing motions induced by the waves. A more realistic relation could also be introduced between the bearing of the target ship and the rudder angle ζ; for example, a "lead" angle could be added so that S would tend to sail towards some future position of T rather than the "present" position.

However, sufficient indication has been given of the ways in which the various computing elements may be combined to solve real problems, and the possibilities of extending the ship simulator will not be pursued further.

5

D.C. AMPLIFIERS

The most important item in simulators of the type described earlier is the high-gain direct-coupled amplifier. This device has already been briefly mentioned, and a fuller account of the principles of operation and of some problems of design will now be given, although a detailed discussion is outside the scope of the present work.

Ideally, the d.c. amplifier should give an output voltage which is an exact magnified image of the input voltage, and in order that a high degree of feedback can be applied by simple means the amplifier should give a reversal of sign between input and output. Thus if the input voltage is v, then the output voltage should be $-Mv$, where M is a positive constant, and the constant ratio of output to input voltage should be maintained if v varies with time. Practical amplifiers fail to give this ideal performance for a number of reasons, chief among which are the effects of non-linearities and of unwanted voltages combining with the desired voltages. The non-linearity appears as curvature of the graph relating input and output voltages, i.e. M does not remain constant when v varies. This curvature tends to increase rapidly as the output voltage approaches the "overload" value, but can usually be reduced to a tolerable level by the use of negative feedback and by restricting the excursion of the output voltage to a range well within the "overload" values.

The effect of unwanted voltages is to give an output voltage

$$V = -Mv + V'$$

where V' is a voltage which bears no relation, or at least no simple relation, to the input voltage. V' arises mainly from two causes, *viz.* "drift" and the effect of the grid current of the first valve.

The ideal requirement that $V = -Mv$ includes the requirement that $V = 0$ when $v = 0$, and d.c. amplifiers are fitted with some device such as an adjustable bias control by which this condition can be set up before computation starts. Such a control is shown at R in fig. 39; but although this control may be carefully set to give "zero out for zero in" initially, the effects of drift and grid current during the computation may cause an error.

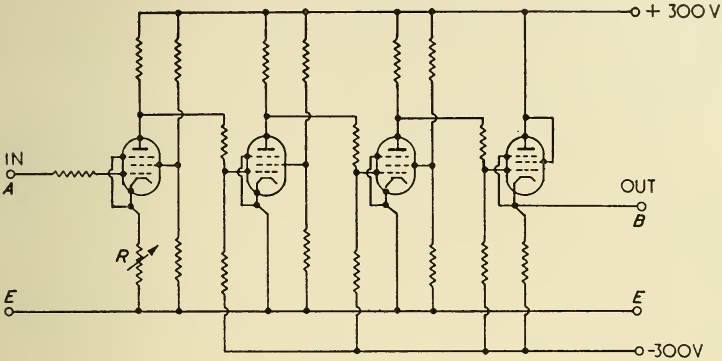


FIG. 39. D.C. Amplifier

5.1 DRIFT IN THE SUMMING AMPLIFIER AND INTEGRATOR

To explain these effects reference will be made to fig. 39, which, although incomplete, illustrates some features common to many d.c. amplifiers. The coupling between each anode and the following grid is by means of a resistor chain, of which the lower terminal is connected to the negative H.T. supply "rail". The values of the resistors are adjusted so that in the absence of an input voltage each grid is at a small negative potential relative to the cathode, equal to the bias required for normal working. For the last stage, which is a cathode follower, the cathode terminal should be at earth potential when there is no input voltage to the amplifier. For the first stage the grid is at earth potential when there is no input voltage, so bias is provided by a cathode resistor.

Before the amplifier is used in a computing circuit the input voltage is reduced to zero by short-circuiting the input terminals, and R is adjusted so that the output voltage is zero. If the setting of R is then left unaltered it will be found after a time that the output voltage will slowly change from zero, even though the input voltage is still zero, and it is this slow departure from the condition of "zero out for zero in" which is called "drift". There are several contributory causes, including ageing and changes of temperature in valves and components, and changes in the high-tension and heater supply voltages. These effects can be minimized by the use of high quality components, suitable choice of valves and operating conditions, careful circuit design, and stabilization of supply voltages, and for some purposes satisfactory performance can be achieved by these means; but for highest accuracy additional devices for reducing drift have been devised, and some of these will be described in Sections 5.4 and 5.5.

The effect of drift in any one stage is equivalent to the introduction of a small voltage, and the magnitude of the drift voltage appearing at the output terminal will depend on both the magnitude of this fictitious small voltage and the amplifier gain between the point where the voltage appears and the output terminal. This means that drift is most serious in the first stage and less important in succeeding stages. It is convenient to refer the drift, wherever it may arise, to the grid of the first valve and to represent the magnitude of the drift by the value of the single voltage, applied at the first grid, which would give the observed output voltage if drift were absent.

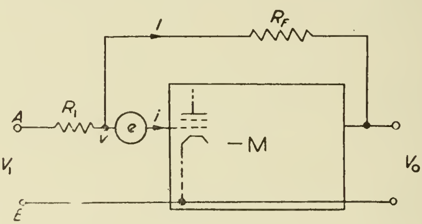


FIG. 40

This device is used in fig. 40, where e is the drift voltage, referred to the first grid, i the grid current, R_F is a feedback resistor, and R_1 is an input resistor.

For this arrangement,

$$\begin{aligned} V_O &= -M(v+e) \\ IR_F &= v - V_O \\ v &= V_1 - iR_1 - IR_1 \\ &= V_1 - iR_1 - \frac{vR_1}{R_F} + \frac{V_O R_1}{R_F} \\ &= \frac{R_F}{R_1+R_F}\left\{V_1 - iR_1 + V_O\frac{R_1}{R_F}\right\} \end{aligned}$$

Hence

$$-\frac{V_O}{M} = \frac{R_F}{R_1+R_F}(V_1 - iR_1) + V_O\left\{\frac{R_1}{R_1+R_F}\right\} + e$$

and if M is very large,

$$V_O \doteqdot -\frac{R_F}{R_1} V_1 + iR_F - e\left\{\frac{R_1 + R_F}{R_1}\right\}$$

$$= -GV_1 - e(1+G) + iR_F \tag{21}$$

where $G = R_F/R_1$ is the effective gain of the amplifier with feedback. In the absence of feedback, and assuming for the present that grid current is zero,

$$V_O = -MV_1 - Me,$$

so that the addition of feedback has reduced the drift component of V_O in the ratio $M:(1+G)$ and the component of V_O due to V_1 in the ratio $M:G$. The fractional error in V_O due to e is e/V_1 when feedback is not used, since

$$V_O = -MV_1\left(1 + \frac{e}{V_1}\right),$$

and the corresponding fractional error with feedback may be shown, by re-arranging equation (21), to be

$$\left(1 + \frac{1}{G}\right)\frac{e}{V_1}$$

so that the addition of feedback has increased the fractional or percentage error, although the increase will probably not be large, since G is not usually a small fraction. It is generally the percentage error which is important, rather than the absolute error, so that although negative feedback is of great value in improving the performance of the amplifier in other ways, it does not have any large effect on the inaccuracy due to drift.

Besides drift, there is in equation (21) a spurious component of output voltage due to grid current in the first valve of the amplifier. In fig. 40 it may be imagined that there is within the valve a generator having one terminal connected to cathode and one to grid, so that grid current flows whenever a continuous external circuit is provided between grid and cathode. If the input terminals AE are short-circuited a current will flow in R_1, and since the output impedance of the amplifier, measured between the output terminals, is not infinite a current will also flow in R_F. Thus the grid will not be at the same potential as if there were no grid current. When the resistor R of fig. 39 is adjusted during the zero-setting operation immediately before the amplifier is used this potential due to grid current will automatically be taken into account, so that at first sight there should be no error from this cause. However, the magnitude of the grid current, and hence of the spurious potential will change as the valve warms up, and this will give a spurious output voltage. The potential due to grid current will also change if R_1 or R_F is changed, but in an amplifier with manual zero-setting this change will be compensated when the zero-set adjustment is next made, provided the total effective resistance between grid and earth during zero-setting is of the same value as when the computing connections are made. In practice, of course, there is no obvious method of determining whether a given spurious output voltage is due to grid current or to drift, but the distinction is important for some purposes, especially in the consideration of some drift-compensation devices.

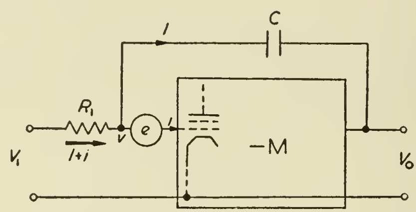


FIG. 41

Amplifier drift and grid current also affect the performance of the amplifier when it is used as an integrator, and because of the integrating action a steady spurious voltage or current at the first grid will give at the output not a steady voltage, but a voltage which increases linearly with time. Thus, in fig. 41,

$$V_O = -M(v+e)$$
$$v = V_1 - IR_1 - iR_1$$
$$I = -(V_O - v)pC$$

Hence,

$$v = V_1 + (V_O - v)pT - iR_1 \quad \text{where } T = R_1C$$

$$= \frac{V_1 + V_O pT - iR_1}{1+pT}$$

and

$$V_O = -M\left\{\frac{V_1 + V_O pT - iR_1}{1+pT} + e\right\}$$

or, if M is very large,

$$V_O \doteqdot -\left(\frac{V_1 - iR_1}{pT}\right) - e\left(\frac{1+pT}{pT}\right)$$

$$= -\frac{V_1}{pT} + \frac{iR_1}{pT} - e\left(1 + \frac{1}{pT}\right)$$

In this equation the first term on the right represents the required integral of the input. The second and third terms represent spurious outputs due to grid current and drift. If i and e are constant these two outputs will increase continuously, and unlike the corresponding errors which occur in a summing amplifier the spurious output due to grid current is not compensated in the zero-setting operation. For zero-setting in an integrator the usual procedure is to replace the capacitor temporarily by a resistor, the capacitor being short-circuited meanwhile to remove any residual change. If a drift voltage e is present at this stage the adjustment of the zero-setter will in effect introduce an equal and opposite potential. When the temporary feedback resistor is replaced by the uncharged capacitor, the drift voltage at the first grid will be unaltered, but the grid current, which was previously divided between the input resistor and the feedback resistor now flows only in the input resistor, so that the spurious grid voltage due to grid current is now different, and the output voltage immediately begins to grow in response to this small input. It is possible to compensate for this effect; for example instead of completely disconnecting the temporary feedback resistor the end connected to the output terminal could be connected to earth instead so that the resistance between grid and earth remained unaltered. In practice, however, the performance of such compensation schemes is often disappointing, because the behaviour of grid current is less simple than has been assumed here. In particular, the internal impedance and e.m.f. of the valve, regarded as a generator of grid current, vary rapidly and non-linearly with changes of grid-cathode potential and cathode temperature.

The usual practical remedy for troubles due to grid current is simply to choose a valve whose grid current is sufficiently small. Fortunately, valves are available which suit most requirements, even though for the highest accuracy it may be necessary to use a valve of the "electrometer" type and accept the consequent reduction in gain due to the lower mutual conductance of such valves.

5.2 THE THREE-STAGE AMPLIFIER

Many practical circuits have been devised for d.c. amplifiers (Ref. 10 and 11), and descriptions will now be given of examples of three important classes which have been used for computing. These are the "three-stage" amplifier with manual zero adjustment; the single-stage amplifier; and the "drift-corrected" amplifier.

To achieve the necessary reversal of sign between input and output most amplifiers use an odd number of stages, and since a single stage gives insufficient gain for many purposes the most popular arrangements use three stages. The circuit diagram of an amplifier of this type which has been used in analogue computers of moderate

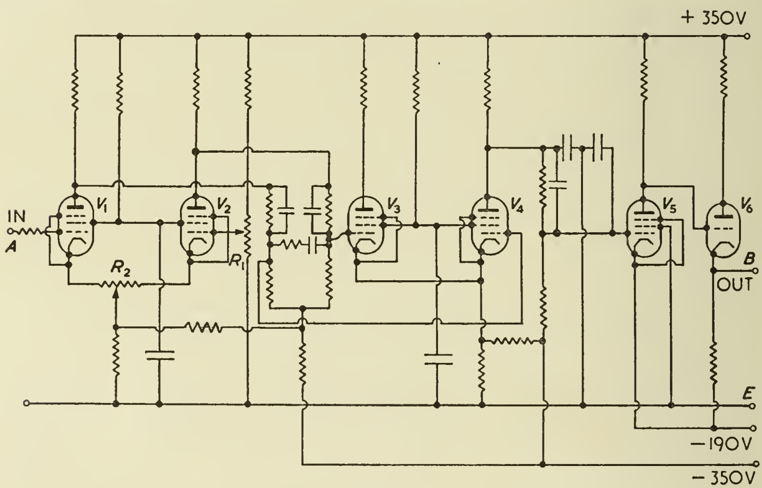


Fig. 42. Three-Stage D.C. Amplifier

accuracy is shown in fig. 42. Compared with the skeleton circuit of fig. 39 the chief differences are the relative complexity of the inter-stage networks, and the use of pairs of valves in push-pull. The resistance chains for $V_1 - V_4$ are basically the same as in fig. 39, but certain additional resistors have been introduced to give some compensation against the effects of varying h.t. supply voltages. The use of pairs of valves is also a compensation device (Ref. 11). V_1 and V_2 form a cathode-coupled pair, sometimes called a "long-tailed" pair, and if a positive voltage is applied to the input terminal the anode current of V_1 increases and that of V_2 decreases. If the heater voltage rises, the anode currents of both valves rise, and the anode potentials fall, so that ideally, with perfectly-matched valves, the difference between the anode voltages does not change. In practice, using a matched pair of "aged" valves, a useful measure of compensation is achieved. The anode voltages from V_1 and V_2 are passed on to V_3 and V_4, which form another long-tailed pair. If equal voltages of the same sign are applied to the grids of V_3 and V_4 these two valves behave as though they were connected in parallel, and the large common cathode resistance gives a large measure of negative feedback so that only a small change of anode current occurs. If, however, equal voltages of opposite sign are applied the anode current of one valve increases and the anode current of the other valve decreases by an equal amount. There is thus no appreciable change of potential across the cathode resistor, and hence no feedback effect, and the magnitude of the anode current changes is about the same as in a normal amplifier stage. Thus the $V_3 - V_4$ stage amplifies normally any difference of potential between the two grids, resulting from a legitimate input signal to V_1 and V_2, but is almost unaffected by potential changes of the same sign which would be caused by change of supply voltage in the $V_1 - V_2$ stage. V_3 is used only as a compensating valve, and V_5 forms a single-valve stage fed from the anode of V_4. The resistance chains are so arranged that the anode of V_5 is at a slightly negative potential relative to earth, sufficient to provide the bias for V_6 which is a cathode follower with its cathode at earth potential. The resistor in the anode circuit of V_6 performs no circuit function, but is inserted to prevent excessive anode dissipation.

The capacitors are added to ensure that the amplifier is stable when feedback is applied. The gain of an amplifier of this type is about 50,000, or 90 decibels, and it is usually desirable to maintain the gain at a roughly constant value at frequencies from zero up to, say, 100 c/s. Above this frequency the gain falls off, but the manner in which it falls off is determined by the need to maintain stability when feedback is applied. Thus suppose the amplifier has equal input and feedback resistors, giving an overall gain of unity. Then the loop gain is 25,000, or 84 decibels, which is half the no-feedback gain since the resistors form, in effect, a two-to-one potential divider in the feedback loop. For satisfactory stability the phase margin (Refs. 12-16) should be not less than about 30°, and this means that the gain cannot be allowed to fall more rapidly than 10 decibels per octave; that is to say that the gain at a given frequency f, expressed as a voltage ratio, must be not less than one third of the gain at a frequency $f/2$. This condition should be satisfied at all frequencies up to a point at which the loop gain has fallen to 0·5, expressed as a voltage ratio (or -6 db), so that the curve representing the fall in gain must be controlled while the gain falls by 90 db, i.e. over a range of nine octaves if the constant slope of 10 db per octave is maintained. Nine octaves above 100 c/s is approximately 50 kc/s, so that the loop gain characteristic must be controlled in shape from zero frequency up to at least 50 kc/s. In practice it is not easy to maintain the slope of the characteristic very near the desired 10 db/octave over the whole range, and regions of lower slope may appear, resulting in a higher frequency for the zero-gain point and a wider frequency range over which the shape must be controlled. If the feedback and input resistors are not equal the loop gain will be somewhat different, and will approach 90 db if R_F is small compared with R_1. Thus it is desirable to control the characteristic shape over a further octave if stability under all likely conditions is required. In some circumstances it may be permissible for the gain to fall more rapidly than 10 db/octave over part of the frequency band.

When the amplifier is used as an integrator, as in fig. 4*a*, then the feedback components will make some contribution to the loop characteristic, and assuming that the voltage V_1 is supplied from a source which has a very low impedance, the effect will be equivalent to introducing the simple network of fig. 43. At frequencies which

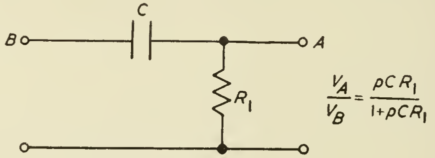


FIG. 43

are comparable with or less than the reciprocal of the time constant CR_1 this circuit will produce some attenuation and phase shift; but the values of CR_1 commonly used are such that at frequencies above, say, 100 c/s the reactance of C is very small compared with R_1 so that in the higher frequency region, where the loop characteristic must be controlled, the integrator components make no contribution to either phase shift or attenuation, and the requirements are the same as when the amplifier is used with a feedback resistor which is small compared with the input resistor.

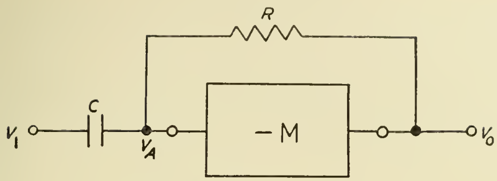


FIG. 44. Differentiator

If, however, the amplifier is used in a differentiator circuit (fig. 44), the effective network introduced is that shown in fig. 45 which has a transfer function

$$\frac{V_A}{V_B} = \frac{1}{1+pRC}$$

At frequencies above 100 c/s and normal values of RC, $pRC \gg 1$, so that

$$\frac{V_A}{V_B} \doteq \frac{1}{pRC}$$

This represents a rate of fall of gain of 6 db per octave (or two to one in voltage ratio for two to one in frequency) and a phase lag of 90°, so that the amplifier proper must have a rate of fall of gain of about 4 db per octave with a phase lag of 60° if the 30° phase margin

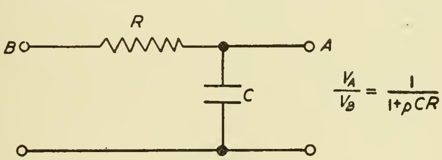


FIG. 45

is to be maintained. There is no fundamental difficulty in achieving this, but it is different from the characteristic required for a summing amplifier or integrator. Thus, while a high-gain d.c. amplifier can be designed which will operate either as a summing amplifier or an integrator, depending only on whether a resistor or capacitor is used as the feedback impedance, it is more difficult to design the same amplifier so that it will also operate as a differentiator. This is one of the reasons why differentiators are not used extensively in electronic computing machines. Another reason will be given in Section 6.7.

Referring again to the circuit of fig. 42 two variable resistors are shown. R_1 is the "zero-set" control, whereby the potential of the output terminal is set to zero when the input terminal A is earthed.

To take some account of grid current in V_1 this temporary earth connection may be made via a resistor of the same value as will appear between the grid and earth when the normal computing circuit is connected. The resistor R_2 is a pre-set control provided to improve the compensating action of V_1 and V_2.

In favourable circumstances, using well-stabilized supplies, with supply mains free from sudden changes of voltage, and with well-aged components running at steady temperature, an amplifier of this type, set for unity gain, may show a drift of 10 mV or less over a period of up to an hour. In other circumstances the drift may be several times larger. The first stage uses a low-grid-current valve, so that grid current effects are not troublesome. The overload output voltage is about ± 70 volts, although for the highest accuracy it is usual to restrict the output to about half this value. The output impedance of the amplifier, connected for an overall gain of ten times, is less than 0·1 ohm, and since the input resistance of a summing amplifier or integrator is not usually less than about 100,000 ohms, the input resistors of a number of amplifiers can be connected in parallel to the output terminal of another amplifier without errors due to loading effects, i.e. without appreciably altering the output terminal voltage.

Amplifiers of the type shown in fig. 42 are thus suitable for use in computing machines of moderate accuracy, especially if the number of amplifiers is small, so that frequent adjustment of the zero-set control is practicable. They can also be used in larger numbers if aid is provided to reduce the delay and labour of zero setting, as in the GEPUS machine described in Chapter 11.

Other versions of the three-stage amplifier have been designed, including some with push-pull output stages. This arrangement has the advantage that both "signs" of the output voltage are available and there is no need to use amplifiers for sign reversing. The resultant economy, however, is often smaller than might be expected, because in many cases, if the block diagram is carefully arranged, the number of reversing amplifiers needed is only a small fraction of the total number of amplifiers. Furthermore, the push-pull arrangement requires the provision of at least one extra valve and the provision and adjustment of an additional feedback component for each amplifier, and it is often considered that the small decrease in the number of amplifiers needed with the push-pull circuit is insufficient to justify the extra components.

5.3 THE SINGLE-STAGE AMPLIFIER

The basic single-stage amplifier is illustrated in fig. 46, and with normal anode and screen potentials is capable of a gain of around 50 times. The output swing is limited because of departure from linearity, and the output impedance is high. Nevertheless, this amplifier is occasionally useful, and has obvious attractions of simplicity, economy, and freedom from instability troubles.

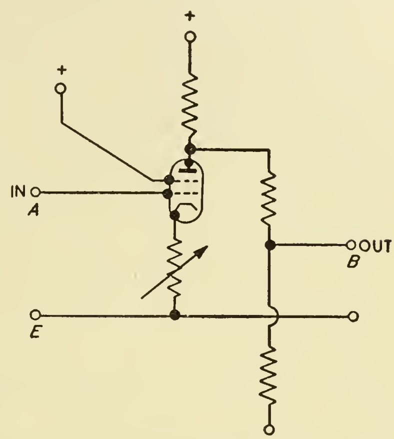


Fig. 46. Single-Stage D.C. Amplifier

A more elaborate "single-stage" amplifier can be formed by adding a cathode follower to the amplifying stage of fig.46 and furthermore the gain of the amplifying stage can be raised if care is taken to adjust the anode and screen voltages and resistor values for maximum gain. This usually involves a low screen voltage so that the anode current is small and a high value of anode resistor can be used

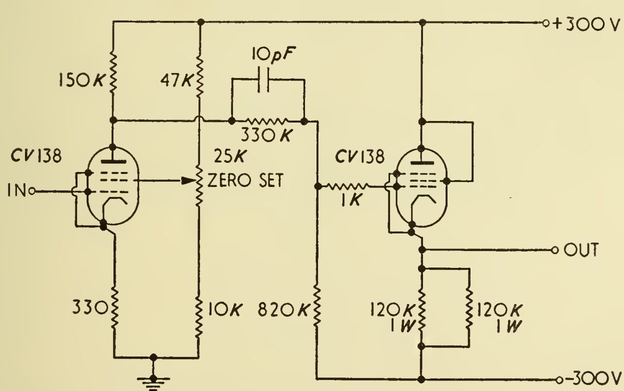


Fig. 47. Single-Stage D.C. Amplifier with Cathode Follower

without excessive voltage drop. Fig. 47 shows a set of values used by D. W. Allen which gives an overall gain of 120 times. The first valve is a steep-slope h.f. pentode, which has a rather high grid current, so that input resistors are preferably kept below 100,000 ohms. Furthermore, the gain is not equal to the ratio of feedback and input resistors, and it is desirable to measure the gain and adjust the resistors accordingly. However, the output impedance is quite low, there is no serious departure from linearity over an output swing of ± 25 volts, and drift troubles are almost entirely absent. Amplifiers of this type have been used successfully in a simulator where errors of a few per cent could be tolerated.

Another simple amplifier has recently been devised by P. E. Benyon and M. T. Hawkins, using two double-triode valves. The first valve is connected as a cathode-coupled stage, and with the input signal applied to the grid of one triode and the output taken from the anode of the other the stage gives normal gain with no phase reversal. One triode of the second valve is used as a normal amplifier, and the other triode as a cathode follower. The amplifier has an overall gain of about 700, with a low output impedance and a useful measure of compensation against drift in the first stage. A number of specimens of double triode (CV 492) have shown grid currents of the order of 10^{-10} to 10^{-11} ampere.

5.4 THE DRIFT-CORRECTED AMPLIFIER

The drift-corrected amplifier is a development of the "three-stage" amplifier in which additional equipment is provided to reduce the drift which is the most objectionable feature of ordinary d.c. amplifiers.

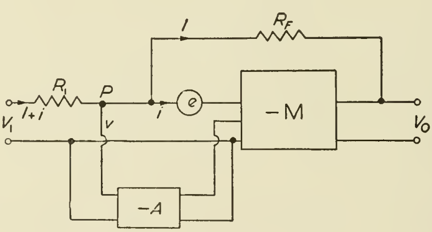


Fig. 48

The basic principles of the drift-corrected amplifier may be explained with reference to fig. 48. This represents a high-gain d.c. amplifier, with input and feedback resistors, and in addition there is a second d.c. amplifier of gain $-A$ which takes as its input voltage the voltage v at the junction of the resistors, and feeds its output voltage into the amplifier M where it is combined with the normal input voltage. Suppose initially that V_1 is zero. Then, neglecting grid current for the moment,

$$V_O = -M(e+v+Av) \tag{22}$$

where e is a drift voltage, and

$$v = \frac{R_1}{R_1+R_F}V_O$$

It is assumed here that the voltage Av is fed into the M amplifier in such a way that the system is stable when A is positive, i.e. when the auxiliary amplifier gives a 180° phase shift.

Eliminating v from these equations,

$$V_O = -Me - M(A+1)V_O\left(\frac{R_1}{R_1+R_F}\right)$$

i.e.

$$V_O\left\{1+M(A+1)\left(\frac{R_1}{R_1+R_F}\right)\right\} = -Me$$

Or, making the usual assumption that M is very large,

$$V_O \doteqdot -\left(\frac{1}{A+1}\right)\left(\frac{R_1+R_F}{R_1}\right)e$$

$$= -\left(\frac{1}{A+1}\right)(1+G)e$$

where, as before, $G = R_F/R_1$.

Comparing this with equation (21) when V_1 and i are both zero shows that the input voltage produced by e has been reduced in the ratio $(A+1):1$, so that if A is appreciably greater than unity a valuable reduction of the effects of drift is possible.

If V_1 is not zero,

$$V_O = -M(e+v+Av)$$

$$v = \frac{R_1}{R_1+R_F}(V_O - V_1) + V_1$$

$$= \frac{R_F V_1 + R_1 V_O}{R_1+R_F}$$

Hence

$$V_O = -Me - M(A+1)\frac{R_F V_1 + R_1 V_O}{R_1+R_F}$$

i.e.

$$V_O\left\{1+M(A+1)\frac{R_1}{R_1+R_F}\right\} = -Me-M(A+1)\frac{R_F}{R_1+R_F}V_1 \tag{23}$$

Or, if M is very large,

$$V_O = -\frac{1}{A+1}(1+G)e-GV_1$$

where $G=R_F/R_1$, so that the output due to the input V_1 is apparently unaffected by the auxiliary amplifier. However, examination of the foregoing equations shows that the effective gain of the amplifier, for the voltage V_1, has been increased from M to $M(A+1)$, so that the errors due to neglecting terms which do not contain M is reduced in the ratio $(A+1):1$, and the benefits of feedback, such as improved linearity, etc., are increased.

If grid current flows from the first valve of the amplifier M, the auxiliary amplifier will not compensate for the consequent spurious output voltage. For assume in fig. 48 that e and V_1 are zero, and that a grid current i flows as shown. Then

$$V_O = -M(v+Av)$$

$$I = -\frac{V_O-v}{R_F}$$

$$I+i = -\frac{v}{R_1}$$

whence

$$v = -\left(\frac{R_1R_F}{R_1+R_F}\right)i+\left(\frac{R_1}{R_1+R_F}\right)V_O$$

and

$$V_O = -M(A+1)\left\{V_O\left(\frac{R_1}{R_1+R_F}\right)-i\left(\frac{R_FR_1}{R_1+R_F}\right)\right\}$$

so that if M is very large,

$$V_O = iR_F \tag{24}$$

This output voltage is independent of the value of A and is unaltered if the auxiliary amplifier is absent, provided M is large. This general result can be deduced directly from fig. 48 if it is assumed that the values of R_1 and R_F are of the same order. Since M is very large v must always be very small, and whenever v is not exactly zero V_O will be very much greater in magnitude and of opposite sign.

Thus when V_1 is zero and grid current flows the current will divide between R_1 and R_F, but because of the appearance of a large V_O when v is not zero the current through R_1 will be only a very small fraction of i and the current through R_F will be very nearly equal to i. As M approaches infinity the value of v must shrink towards zero if the amplifier does not overload, so the current through R_1 also tends to zero and all of i flows through R_F, giving $V_O = iR_F$.

When the amplifier is used as an integrator, the auxiliary amplifier gives similar benefits so far as reducing the effect of drift and increasing the loop gain are concerned, but again there is no reduction of the error due to grid current. When the input voltage V_1 is zero a current very nearly equal to the grid current must flow through the integrator capacitor, corresponding to the flow of almost all the grid current through the feedback resistor in fig. 48. By substituting $1/pC$ for R_F in the equations leading to equation (24) it may be shown that the output voltage due to grid current is given by

$$V_O = i/pC.$$

For a steady value of i this means that V_O must change continuously. However, as mentioned earlier, valves are available which have grid currents so small that the grid current error can be made acceptably small.

In some circumstances, especially if the input capacitor of amplifier A has a low insulation resistance, it may be necessary to take account of the grid current due to the first valve of this amplifier.

The use of the auxiliary amplifier reduces amplifier drift for both summing amplifier and integrator connections, and also enhances the linearizing and other beneficial effects of negative feedback by increasing the effective amplifier gain from M to $M\ (A+1)$. However, the foregoing demonstration of these effects of the auxiliary amplifier has been based on the assumption that this amplifier was itself free from drift. Inspection of fig. 48 shows that if a drift voltage e' appears at the input terminal of amplifier A, then the term Av in equation (22) is replaced by $A(v+e')$ and the output voltage is then

$$V_O = -\frac{1}{A+1}(1+G)(e+e'A)$$

Assuming that $A \gg 1$, this means that an additional voltage $e'A/(1+A) \doteqdot e'$ appears with the output voltage, indicating that amplifier A has no useful effect in reducing this drift voltage. If the benefits from the auxiliary amplifier are to be realized, therefore, it is essential that the amplifier A be drift-free.

The usual method of removing, or at least greatly reducing the effects of drift in the auxiliary amplifier is to use the input signal to modulate a carrier signal which is amplified, and then demodulated or rectified before being applied to amplifier M. The usual modulator and demodulator circuits, using either thermionic or metal rectifiers

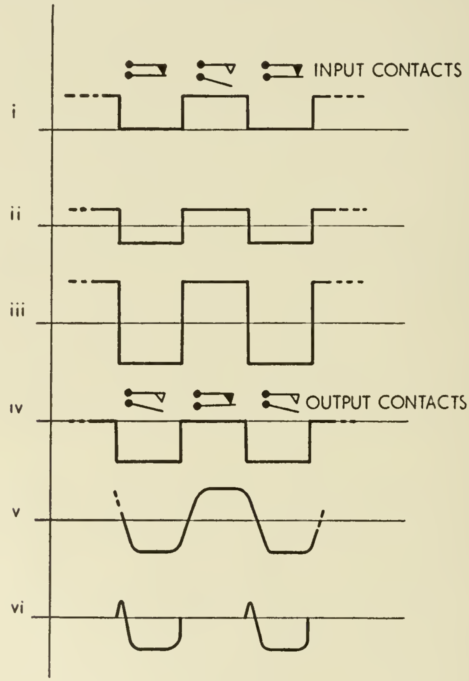


Fig. 49

are not very suitable in this application because the residual potentials due to contact potentials and to unbalance between pairs or sets of valves or rectifiers tend to mask the small input potentials and to give spurious output voltages which bear little or no relation to the voltages it is required to amplify. Considerable success has, however, been achieved with a mechanical modulating device in the form of high-speed relay forcibly vibrated by an alternating current of a suitable frequency. One contact of the relay is earthed and the other is connected to the input terminal of amplifier A, a resistor being connected between this terminal and point P (fig. 48) to prevent short-circuiting of the input signal to amplifier M. Thus when the relay is energized with alternating current there appears at the input terminal of A a square-wave voltage which alternates between zero and the voltage at point P. The alternating component of this square wave provides a measure of the voltage at P, so the direct component can be discarded and amplifier A need only be capable of amplifying alternating potentials. The amplified voltage must be rectified and smoothed before being applied to the main amplifier, and the rectification is again usually performed mechanically, although the objections to thermionic or metal rectifiers are now less strong since any residual potentials will effectively combine with the drift of amplifier M and will consequently be reduced by the action of amplifier A. The rectification can be performed by simply short-circuiting the output terminals of the amplifier for alternate half cycles, provided the contacts can be closed at the beginning of a half cycle and opened at the end. This requirement can be met by using a second pair of contacts on the relay used for "chopping" the input signal. Assuming that the relay has two pairs of contacts, one pair of which opens when the other pair closes, the waveforms at various points in the circuit are illustrated by fig. 49. Wave (i) is the chopped input wave appearing at the input terminal of amplifier A. Wave (ii) is the alternating component of (i), and (iii) is the amplifier output, drawn to a reduced scale. The action of the output contacts is to give wave (iv), which has a direct component proportional in magnitude to the original input voltage, but of opposite sign. This is consistent with the previous assumption that the auxiliary amplifier gives a reversal of phase.

The waveform (iii) assumes that the amplification is free from distortion, but in practice there will be some change of the waveshape and possibly also a phase-shift. The wave (v) shows the output wave of the amplifier when there is some attenuation of the higher-frequency components of the input wave and a phase lag. Both these faults are shown in greater magnitude than would be expected in practice, but even so the mean value of the rectified wave (vi) is still proportional to the amplitude of the input wave. The mean value is, however, somewhat less than that given by wave (iv), i.e. the effective gain of the amplifier is reduced. This is not of great importance provided the distortion of waveform and the phase lag remain reasonably constant.

Although in fig. 49 the input and output contacts are shown as separate pairs, one contact of each pair is connected to earth, and it is possible to replace the two pairs by a single change-over, as shown in fig. 50. This is the arrangement usually adopted in practice, since it eases the design of the relay.

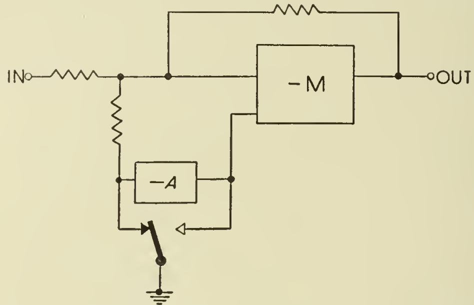


FIG. 50. Drift Correction using "Chopper" Relay

The filter which is used to smooth the rectified output of the amplifier should be capable of reducing the residual ripple to about the same value as the drift voltages expected at the input of amplifier M, otherwise there will be objectionable ripple at the output of M. The high attenuation required can be achieved by using a filter with a number of sections, and having a cut-off frequency say within an octave or two of the chopping frequency, but this leads to stability difficulties. For if equation (23) is re-written with $e=0$, the overall gain is given by

$$\frac{V_O}{V_1} = -G\left\{\frac{1}{1+\dfrac{1+G}{M(A+1)}}\right\} \tag{25}$$

which is very near the nominal value G; but from the stability point of view the remarks made earlier (Section 5.2) regarding the need to control the decrease of gain of the amplifier M now apply to the total effective gain $M(A+1)$, where A is a vector quantity representing both the amplitude and phase response of the auxiliary amplifier, including the filter. If the filter is of the resistance-capacitance type, each section will give a phase shift which approaches 90° as the frequency rises towards cut-off and remains close to 90° at all higher frequencies, so that even two sections give a phase-shift approaching 180°. Thus if the arrangement is to be absolutely stable (i.e. not conditionally stable in the Nyquist sense) (Refs. 12 to 16) the filter should have no more than two sections, and in order to achieve adequate suppression of the alternating component using only two sections the cut-off frequency must be made much lower than the chopping frequency. At frequencies well above the "cut-off" (taken to be equal to $1/2\pi$ RC), two sections of RC filter give an attenuation which increases at 12 db/octave (four-to-one fall in voltage ratio for two-to-one increase in frequency), so that for example an attenuation of 72 db, or a voltage ratio of 1 : 4,000, is obtained at a frequency equal to 2^6 or 64 times the cut-off frequency. Owing to the need for controlling the shape of the feedback loop characteristic it may not be practicable to use two sections of equal time constant, and so the filtering action is somewhat less efficient than these figures indicate. An inductance-capacitance filter could, of course, be used, but a single section of low-pass filter also gives a slope of 12 db per octave and a phase shift of $-180°$ at frequencies above cut-off. Furthermore, there is less possibility of controlling the characteristic, and inductances are less convenient than resistors, so the RC type of filter is usually chosen. In practical designs the gain of amplifier A is often around 500 times, and the chopping frequency is not usually higher than about 200 cycles per second because of the difficulty of securing reliable operation of the chopper relay at higher frequencies. Thus, assuming that an attenuation of a few thousand times is required the cut-off frequency of the filter cannot be higher than about 2 c/s, so that the combination of amplifier, relay contacts, and filter form, in effect, a d.c. amplifier having a bandwidth from zero up to about 2 c/s.

The object of using the auxiliary amplifier A in conjunction with the main amplifier M is to give an amplifier of low drift. But since the scheme is only successful if A itself has a low drift the proposed solution seems at first sight to beg the question, and it is natural to enquire why amplifier A, with its choppers and filter cannot be used alone as the main amplifier, since it behaves as a d.c. amplifier of low drift. The main objection to this lies in the small bandwidth imposed by the filter. Problems in dynamics of the kind described earlier commonly include motions which have components of frequencies up to perhaps 10 or 20 c/s, and amplifiers used in computers to handle such frequencies must have bandwidths greater than this if reasonable accuracy is to be achieved (Refs. 17 and 18). A further objection is that the gain of the amplifier is too low for use as a main computing amplifier. There is, of course, no difficulty in increasing the gain above the value of 500 quoted earlier, but any increase in gain requires a corresponding increase in the attenuation provided by the filter, and assuming that the chopping frequency cannot be increased, the cut-off frequency of the filter must be still further reduced.

The narrow bandwidth also means, of course, that the benefits due to using the auxiliary amplifier are realized only over this narrow band. So far as drift correction is concerned, however, this restriction is unimportant since the drifts occurring in amplifier M are slow, and have no appreciable component beyond the filter cut-off frequency. The secondary benefits such as improvement of linearity are lost above the cut-off frequency, but this again is not serious, because with the degree of feedback normally used the performance of the amplifier M is usually adequate in all respects except drift.

As the gain of A falls off the combined gain $M(A+1)$ also falls, but equation (25) shows that when A falls below unity the effective gain tends towards M, and does not fall below M no matter how small A may become. This is an important consequence of the particular method of connecting the amplifiers together. If an ordinary

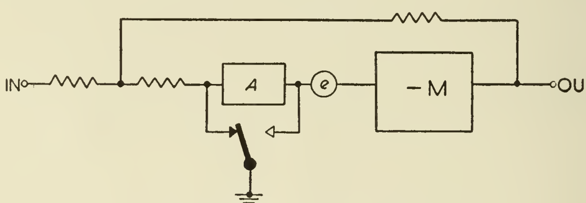


Fig. 51. Amplifiers in Tandem

tandem connection were made, as in fig. 51, the effective reduction of drift voltage e appearing at the input to M would be achieved as before, but the combined gain would be AM, and the bandwidth of the combination would be that of A. In particular, at frequencies where the gain of A was less than unity the combined gain would be less than M.

The earliest proposals for a drift-corrected amplifier of the type described above were published in the U.S.A. in 1948 (Refs. 19, 20), although a different method had been described by Prinz in 1947 (Ref. 21). Several other workers have contributed subsequently (Refs. 22, 23). Following the proposals made by Goldberg (Ref. 22) a drift-corrected amplifier was developed by LANGE, BURT and HOLBOURN, and has been further developed by Elliott Bros. Research

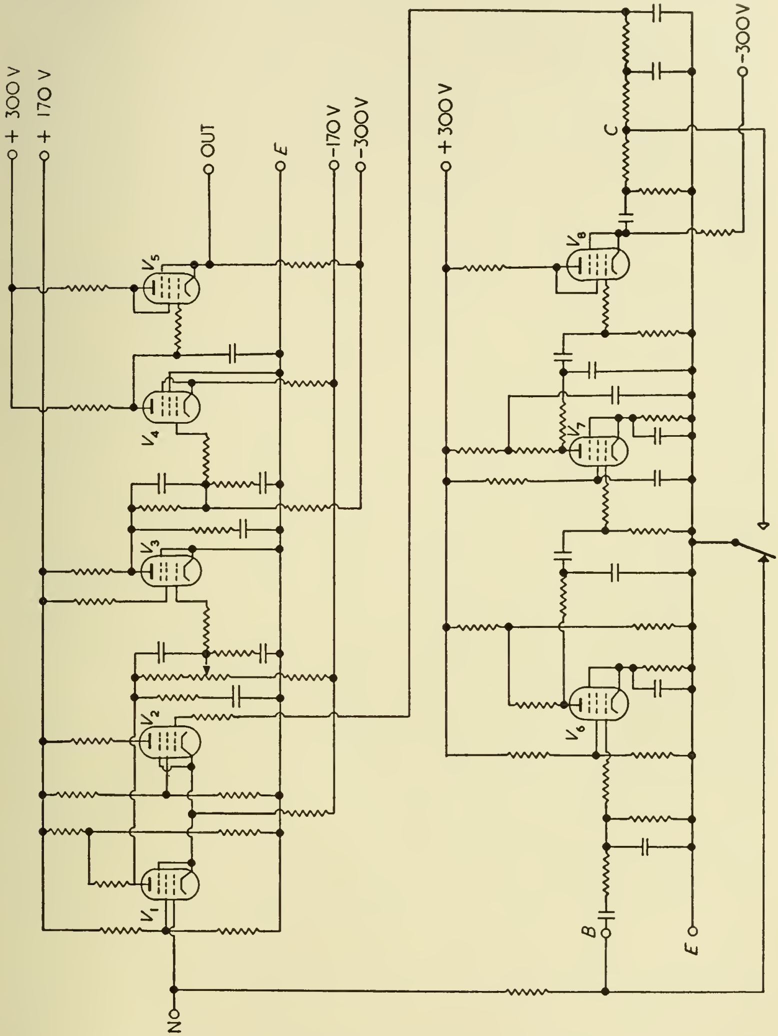


FIG. 52. Drift-Corrected D.C. Amplifier

Laboratories in a form suitable for use in the large TRIDAC computing machine (Section 11.3). The main features of the circuit diagram of this amplifier are shown in fig. 52. The "*M*" section of the amplifier, comprising V_1 to V_5 is basically a "three-stage" amplifier having a pair of valves V_1, V_2, in the first stage, and a cathode-follower output valve V_5. The use of two valves in the first stage gives some measure of compensation against drift, as in the case of the amplifier of fig. 42, and it also provides a convenient means of introducing the output signal from the auxiliary amplifier. The normal input voltage is fed to the grid of V_1 and the output from the auxiliary amplifier is fed to the grid of V_2, so that addition occurs as required, but coupling between the two grids, which might cause instability, is provided only by stray capacitances and can be kept very small.

The first valve is a type CV 432, which is designed to give very low grid current. With the particular working conditions chosen in this amplifier the current is of the order of 10^{-11} ampere, so that even with an input resistor of a few megohms the spurious input voltage due to grid current is a small fraction of a millivolt. The remaining valves V_2 to V_5 are of type CV 138. If the amplifier is required to feed a low-impedance load, drawing more current than can be supplied by a CV 138, an additional cathode-follower stage, using a valve type CV 450 is used, or alternatively a larger valve can be used for V_5, with appropriate changes in the values of the associated resistors. A potentiometer is provided between V_2 and V_3, so that the amplifier can be set to give approximately zero output voltage when the input is zero and the grid of V_2 is earthed. It is a pre-set control which needs adjustment only during the final stage of manufacture, or perhaps when a valve or component is changed.

The auxiliary amplifier, which includes valves V_6 to V_8 is a straight-forward resistance-capacitance coupled amplifier with two amplifying stages and a cathode-follower output stage. The output of the cathode-follower is passed through a filter and then to the grid of V_2. In a later design the auxiliary amplifier has been simplified by using a double triode in place of V_6 and V_7 and omitting the cathode-follower. This has reduced the gain somewhat, but the performance is still satisfactory. The relay contacts are arranged so that the input terminal B and the output terminal C of the auxiliary amplifier are alternately connected to earth, to give the chopping and rectifying action.

The chopper relay used in this amplifier is manufactured by Elliott Bros. to a design by the Radio Corporation of America (R.C.A.) evolved specially for use in chopping devices. It is a non-polarized relay, so that the armature is attracted on both negative and positive half cycles, and the chopping frequency is double the frequency of the current used to energize the driving coil. It will operate over a range of frequencies, and in the present amplifier it is fed with current at 200 c/s, giving a chopping frequency of 400 c/s.

The gain of the M or d.c. section of the complete amplifier is about 60,000 at frequencies up to 100 c/s, falling to about 30,000 at 300 c/s and 10,000 at 1,000 c/s. The phase margin is nowhere less than 30°. The gain of the A or a.c. amplifier is about 8,000 between the grid of V_6 and the anode of V_8, but losses due to the input network and to the rectifying action reduce the effective d.c. gain to about 800. When the two amplifiers are connected together as indicated in fig. 52, and external input and feedback resistors of equal value are connected to give an amplifier of nominally unity gain the measured gain is within $1{\cdot}000 \pm 0{\cdot}001$ at all frequencies from zero to 500 c/s, and the phase shift over this range is nowhere greater than $0{\cdot}1°$. For output voltages between ± 30 V the output is proportional to the input within $0{\cdot}1\%$ when an external load of 7,500 ohms is connected. The linearity is better than this, if, as is usual, the total load resistance is greater than 7,500 ohms, and the departure from linearity can be kept within the limits of $\pm 0{\cdot}1\%$ over a wider range of output voltages if a larger valve than the CV 138 is used in the output stage. When the input terminal is earthed, the voltage at the output terminal remains between about 100μV and 500μV indefinitely, provided that all supplies are well stabilized and the chopper relay is operating correctly. There appears also at the output terminal a residue of ripple from the chopper, at 400 c/s and harmonic frequencies, and some pick-up from the heater supply of the A amplifier, giving altogether an R.M.S. level of about 5mV. This ripple is not usually objectionable, since it does not affect the accuracy of computation of succeeding elements of a simulator, and it can be removed by filtering, if necessary, before the output voltage is passed to an indicating or recording instrument.

5.5 OTHER DRIFT-CORRECTION METHODS

Besides the method of drift-correction used in the amplifier described above several other methods and variations have been described. The method of Prinz (Ref. 21), although it makes use of some type of chopper, is fundamentally different from that described above, and it does not appear to have been widely used. Arrangements for connecting the d.c. and a.c. amplifiers effectively in parallel rather than in tandem have been devised and BUCKERFIELD (Ref. 24) has devised the "Parallel-T" amplifier in which an a.c. amplifier with a chopper is used to cover the range from zero up to some changeover frequency, beyond which a conventional a.c. amplifier comes into operation. These parallel arrangements aim at giving a high and fairly uniform gain over a wide frequency band, and although suited to some applications involving the amplification of small potentials, they seem to offer no advantages for use in analogue computing devices of the type discussed in the previous section. A drift-corrected amplifier similar in principle to that described above has been devised by SUMMERLIN (Ref. 23), who used a chopping frequency of 50 c/s. His system includes a device which enables a single a.c. amplifier to act as the "auxiliary" amplifier to a number of d.c. amplifiers, giving some economy in equipment at the expense of less frequent corrections of drift, though the correction is still adequate for some purposes.

Although the chopper-corrected amplifier can be made to give a good performance the relay itself is still not completely satisfactory. The initial design is difficult, its working life is limited, and it needs careful adjustment. It is therefore natural that attempts should be made to replace it by some non-mechanical device. Modulators and demodulators using dry rectifiers are well known, but, as Summerlin says, they are subject to drift errors of the same type as they are intended to avoid. However, LANGE and LONERGAN have made drift-corrected amplifiers using germanium diodes as modulators and demodulators in which the drift remained less than 10mV for short periods, and there seems to be some hope that this performance could be improved.

An alternative method, which has been developed to the production stage by Elliott Bros., makes use of a "magnetic" modulator and demodulator. Although this device requires careful design and initial adjustment, and although the drift it gives is greater than that given by the relay chopper, it will operate for very long periods and has no mechanical parts to wear, so that it is attractive for applications which can tolerate larger drifts than are given by the chopper relay. Drift correctors of this type also are used in TRIDAC.

The magnetic modulator has two small magnetic cores, shielded from each other and from external magnetic fields, and each core carries three windings. For convenience the windings on one core may be called $1A$, $2A$, $3A$, and those on the other core $1B$, $2B$, $3B$.

Windings $1A$ and $1B$ are connected in series and supplied from a constant a.c. source. Windings $2A$ and $2B$ are also connected in series and the d.c. input is applied to this pair. Windings $3A$ and $3B$ are connected in series and provide the output signal. The windings are so connected and balanced that when no d.c. is applied the e.m.f.s induced in coils $3A$ and $3B$ are equal and opposite, so that there is no net output. When a d.c. signal is applied each magnetic core is subjected to a steady magnetizing force in addition to the alternating force and the e.m.f.s induced in the coils $3A$ and $3B$ are no longer equal and opposite at all times. There is thus an output signal, composed of even harmonics of the excitation frequency, and the amplitude of this signal is a measure of the amplitude of the applied d.c. signal. The harmonic signal can be amplified and rectified in the usual way. An account of a study of a magnetic "inverter" of this type has been given by FROST-SMITH (Ref. 25).

6

COMPUTING WITH PRACTICAL AMPLIFIERS

Some indications have already been given, in Section 1.4, of ways in which high-gain d.c. amplifiers can be used for sign-reversing, summing, and integrating, but it was assumed there that the amplifiers had ideal characteristics. Some consideration will shortly be given to the effects of practical shortcomings, but before this it will be useful to examine other circuits which can be used for summing and integrating with high-gain d.c. amplifiers. Examination of these alternatives will show why the circuits already mentioned are preferred and will also help in understanding the operation of these circuits.

6.1 SUMMATION BY NETWORKS

A voltage proportional to the sum of two or more voltages can be produced by means of the circuit shown in fig. 53. If the current flowing through terminal A is zero,

$$\frac{V_1 - V_A}{R_1} + \frac{V_2 - V_A}{R_2} + \frac{V_3 - V_A}{R_3} - \frac{V_A}{R_A} = 0$$

or,

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = V_A\left(\frac{1}{R_A} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

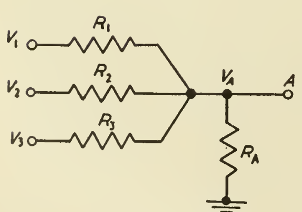


FIG. 53.
Summation Network

If R_A is very small compared with R_1, R_2, and R_3,

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = \frac{V_A}{R_A} \tag{26}$$

very nearly,

and

$$V_A = a_1 V_1 + a_2 V_2 + a_3 V_3$$

where

$a_1 = R_A/R_1$, $a_2 = R_A/R_2$, $a_3 = R_A/R_3$.

Thus the circuit operates as a summing circuit provided R_A is sufficiently small; but although V_A then bears the proper proportional relation to V_1, V_2, and V_3, its absolute value is small. To remedy this, a high-gain amplifier may be connected as shown in fig. 54. Then

$$V_O = -M(a_1V_1+a_2V_2+a_3V_3) \tag{27}$$

If the output and input voltages are to be of the same order of magnitude then Ma_1 etc., must be of the order of unity. The error

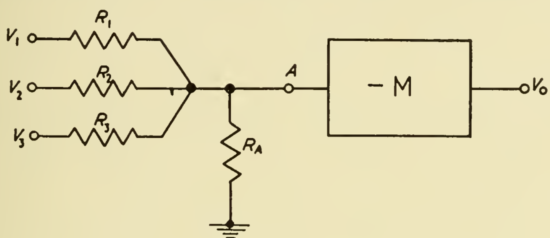


FIG. 54

due to the approximation in equation (26) can be made as small as desired by using a sufficiently high value of M and a correspondingly small value of R_A. This arrangement is sound in principle, and would work well if a suitable amplifier could be provided. However, the output voltage V_O is now directly proportional to M, so that only very small variations in the value of M can be permitted if reasonable accuracy of computation is to be achieved. The two requirements of high value of M and high constancy of this value are not easy to satisfy simultaneously in a practical amplifier, and it is natural to enquire whether negative feedback can be used to

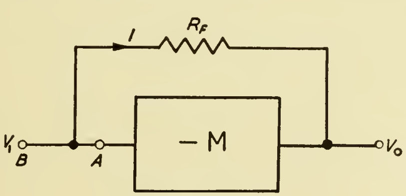


FIG. 55

improve the constancy of gain. The simplest feedback arrangement is that shown in fig. 55, and although this does not help to stabilize the value of M directly it is beneficial in a somewhat unexpected manner. Assuming that V_1 is supplied from a zero-impedance source the voltage at the input terminal A is equal to V_1, the output voltage is $V_O = -MV_1$, and the overall gain has not been altered by the addition of the resistor R_F. However, since there is now a potential difference across R_F there must be flowing in it a current

$$I = \frac{(V_1 - V_O)}{R_F} = \frac{V_1(1+M)}{R_F}$$

The current at A is still zero, so the current must flow via the input terminal B. Thus, when a voltage V_1 is applied a current I flows, so the circuit must now present a resistive input impedance of value

$$Z = \frac{V_1}{I} = \frac{R_F}{1+M} \tag{28}$$

Now it was pointed out above that MR_A/R_1 in fig. 54 should be of the order unity, i.e., R_A should be of the same order as R_1/M. Assuming that R_F in fig. 55 is of the same order of magnitude as R_1 in fig. 54, then by equation (28) Z is of the same order as R_1/M, i.e. of the same order as R_A. Hence, it would be possible to remove R_A in fig. 54 and provide the necessary low resistance by adding a feedback resistor to the amplifier, as in fig. 3*a*.

Now from equation (27),

$$\begin{aligned} V_O &= -M\left(\frac{V_1R_A}{R_1} + \frac{V_2R_A}{R_2} + \frac{V_3R_A}{R_3}\right) \\ &= -MR_A\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right) \end{aligned} \tag{29}$$

Thus some variation of the value of M would be permissible if R_A could be varied in a corresponding manner so as to keep the product MR_A constant. If R_A were an actual resistor any such variation would be impractical, but if R_A is replaced by the input impedance Z of the amplifier of fig. 55 a close approximation to the desired variation is achieved. Equation (29) shows that if the value of M increases, R_A is required to decrease, and equation (28) shows that Z changes with M in the required direction. More precisely, if $R_A = Z$, equation (28) gives

$$MR_A = ZM = \frac{MR_F}{1+M}$$

so that provided M is high,

$$\frac{M}{1+M} = 1, \text{ very nearly,}$$

and $$MR_A = R_F = \text{constant,}$$

so that V_O in equation (29) is very nearly independent of M, provided M always has a high value.

6.2 INTEGRATION BY NETWORKS

A corresponding analysis can be given for the integrator For the circuit of fig. 56,

$$V_C = \frac{1}{C}\int I.dt$$

$$I = \frac{(V_1 - V_C)}{R}$$

so that
$$V_C = \frac{1}{T}\int V_1.dt - \frac{1}{T}\int V_C.dt \tag{30}$$

where
$$T = RC.$$

Thus V_C is proportional to the time integral of V_1, as required, provided the time integral of V_C is negligibly small compared with the time integral of V_1; i.e. provided V_C is negligible compared with V_1. If T is made very large, and if a high gain amplifier is connected after the circuit of fig. 56 to give an output voltage V_O of the same order as V_1, the requirement $V_C \ll V_1$ will be met, since V_C will be of the order of V_1/M. However, the amplifier will again need to have a high and constant value of M, so as before, a feedback arrangement is considered. In fig. 57, the input current is equal to the current flowing in the capacitance, which is

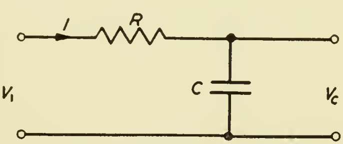


FIG. 56

$$I = \frac{C.d(V_1 - V_O)}{dt} = C(1+M)\frac{dV_1}{dt}$$

since
$$V_O = -MV_1.$$

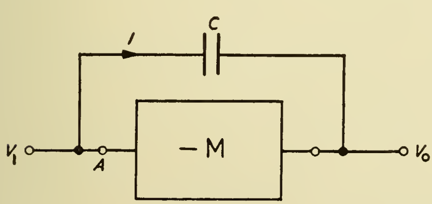


FIG. 57

Now if a voltage equal to V_1 were applied to a capacitance C', the current flowing would be

$$I' = C'\frac{dV_1}{dt}$$

Hence the arrangement of fig. 57 has a capacitive input impedance of the same value as a single capacitance of value $C' = (1+M)C$. Any change in the value of M, besides changing the degree of amplification of the voltages arriving at terminal A of the amplifier, also produces a change in the value of the input capacitance which almost exactly compensates.

The replacement of the capacitor C of fig. 56 by the arrangement of fig. 57 produces the arrangement of fig. 4*a*, the so-called "Miller" integrator. The mechanism by which the input capacitance of the amplifier in fig. 57 appears to have a value $(M+1)$ times the capacitance between the output and input terminals is exactly similar to the undesirable effect which increases the input capacitance of an amplifying valve by an amount equal to the grid-anode capacitance multiplied by the stage gain (Ref. 16). This effect was described by MILLER (Ref. 26) and has since been described by his name. The similarity of the effect occurring in fig. 57 has led to the arrangement being called the Miller integrator, although it is very doubtful whether Miller foresaw any such useful application, and the credit is probably due to BLUMLEIN (Refs. 27 and 28).

It is important to appreciate that the time constant of integration of fig. 4*a* is $T = RC$, and is substantially independent of the value of M provided M is very large. The expression "time constant" used in this way has not the same significance as when used in connection with the circuit of fig. 56. A "time constant of integration" is merely a parameter which appears in equations such as (30) and has no useful relation with the time for the output voltage to reach a fraction $(1 - 1/e)$ of its final value.

6.3 IMPERFECT REVERSING AND SUMMING AMPLIFIERS

Turning now to the effects of the imperfections of practical amplifiers, consider first the sign-reversing amplifier, which may be regarded as a special case of the summing amplifier. The ideal high-gain amplifier for this application would have infinite gain, zero output impedance and infinite input impedance, and exact proportionality between the infinitesimal input voltage and the output voltage for all values of these voltages. The most serious shortcoming of the practical amplifier is drift, which has already been discussed. The next imperfection is that the gain is finite, and it is important to determine how great the gain must be in order to justify the approximations depending on the gain being very large. The relation between input voltage and output voltage for the arrangement of fig. 2 is given by equation (2):

$$\frac{V_O}{V_1} = -\frac{R_F M}{R_F + R_1(1+M)} = -\frac{R_F}{R_1 + \frac{1}{M}(R_1 + R_F)}$$

If M is infinite, then V_O/V_1 has the value $-R_F/R_1 = -G$, say, which is the nominal gain of the amplifier with feedback. If M is large but not infinite the gain with feedback is equal to

$$-G' = -\frac{R_F}{R_1 + \frac{1}{M}(R_1 + R_F)} \tag{31}$$

$$= -\frac{R_F}{R_1\left\{1 + \frac{1}{M}(1+G)\right\}}$$

whence $$G' = G\left(1 - \frac{1+G}{M}\right) \text{ very nearly.}$$

Thus, if G' is not to differ from G by more than, say 0·1%, then the minimum value for M is

$$M = 1{,}000(1+G).$$

In normal computations it is unusual to use values of G greater than about 10, since larger values give rise to voltage swings which tend to overload output stages or else need unduly small input voltages. A value of M of at least 10,000 is therefore desirable. If errors greater than 0·1% can be accepted a smaller value of M will suffice, but in practice it is often found that no great economy can be achieved as a result of reducing the degree of precision demanded unless errors of the order of a few per cent are permissible. The reason for this is that in order for the amplifier to be stable when feedback is applied it must provide a reversal of phase between input and output voltages, at least in the working range of frequencies from zero upwards. This requirement can be met by using an odd number of amplifying stages, and the usual choice is between a single-stage amplifier, giving a value of M of 150 or less, and a three-stage amplifier which can be designed without great difficulty to give a gain of about 50,000.

With $M = -50$ the error between actual and nominal gain G' and G is about 4% even if G is unity, and the error is over 7% if $M = 150$

and $G=10$. These errors can be reduced if, instead of taking $G=R_F/R_1$, the amplifier gain is measured and the values of input and feedback resistors are adjusted to give the desired gain, or if the appropriate values of the resistors are calculated from equation (31). In small computing machines this procedure is sometimes acceptable if there are no other objections to the low gain. For larger machines, however, this method of setting the gain is inconvenient, and as will be seen later, it is still more inconvenient in the summing amplifier. A high value of M is usually desirable for other reasons, and it may be assumed that in the larger computing machines the value of M will be of the order of 20,000 or greater, so that errors due to inequality of G' and G will be a fraction of 0·1%.

If the amplifier is being used as a summing amplifier to add, say, three voltages V_1, V_2, V_3 applied through input resistors R_1, R_2, R_3 with a feedback resistor R_F, the output voltage V_O is given by equation (4):

$$\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3} = -V_O\left\{\frac{1}{R_F}+\frac{1}{M}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_F}\right)\right\}$$

If $R_F/R_1=G_1$, $R_F/R_2=G_2$, $R_F/R_3=G_3$,

$$V_O = -\frac{G_1V_1+G_2V_2+G_3V_3}{1+\frac{1}{M}(G_1+G_2+G_3+1)} \qquad (32)$$

Hence, the effective gain between V_1 and V_O is equal to

$$-G_1' = -\frac{G_1}{1+\frac{1}{M}(G_1+G_2+G_3+1)}$$

and there are corresponding expressions applying to V_2 and V_3. Thus, in the unlikely event of G_1, G_2, G_3, all having values around 10, a value of M about 30,000 would give errors of 0·1%, so that again in most large computing machines errors of this type are likely to be less than 0·1%.

The above considerations of the desirable value of M apply most obviously when M is a positive number, but in practice the amplifier will give phase shifts at the higher frequencies, so that M should properly be regarded as a "vector", i.e. as a quantity having both magnitude and an associated phase angle. Any phase shift in M will, in general, give a phase-shift between input and output voltages, but it can easily be shown that if the phase-shift in M is as much as 90°, provided the magnitude of M is still a few thousand, the phase shift between input and output voltages with normal gains will be less than 0·1°. Furthermore, an amplifier such as that shown in fig. 52 will give phase-shifts of only about 40° at 100 c/s, and correspondingly less at lower frequencies, so that errors due to phase-shift in the amplifier are unlikely to be significant.

In discussing the value of M it has been implied that it was desirable to make the effective gains equal to the ratio of two resistors, e.g. $G_1' = G_1 = R_F/R_1$. As in the case of the reversing amplifier, however, it is possible to achieve a desired ratio between output and input voltages even when M is not large by suitable choice of resistance values.

Equation (32) may be re-written:

$$V_O = -G_1'V_1 - G_2'V_2 - G_3'V_3,$$

where

$$G_1' = \frac{\dfrac{R_F}{R_1}}{1 + \dfrac{1}{M}\left(\dfrac{R_F}{R_1} + \dfrac{R_F}{R_2} + \dfrac{R_F}{R_3} + 1\right)} \tag{33}$$

and there are corresponding equations for G_2' and G_3'. Thus when M is not large the gain between V_1, say, and V_O depends not only on R_F and R_1 but also on R_2, R_3, and M. When values of M, G_1', G_2', and G_3' are known and R_F has been chosen the values of R_1, R_2, and R_3 can be found by simultaneous solution of equation (33) and the corresponding equations for G_2' and G_3'. This is a tedious calculation, and furthermore if any one of the quantities G_1', G_2' or G_3' is changed then new values for all three input resistors must be calculated, and this is unacceptable for a flexible computer.

6.4 OUTPUT LIMITATIONS

Another respect in which the practical amplifier fails to satisfy the requirements laid down for the ideal amplifier is that the available output voltage is limited. As the input voltage rises steadily from zero the output voltage rises also, and initially the two voltages are proportional, but as the rise continues the output increases less rapidly than it would in a perfectly linear amplifier. Thus the gain of the amplifier, defined for this purpose as the ratio of the rate of increase of the input voltage to the rate of increase of the output voltage, decreases as the output voltage increases. The decrease of gain is inappreciable over what is usually called the "working range" of the amplifier, and examination of equation (31), for example, shows that if M has a value of, say, 30,000 a decrease of 20% or even 50% on this value will probably not be serious. However, as the output voltage reaches the "overload point" the gain begins to fall very rapidly, and even the high degree of feedback normally employed in summing or sign-reversing amplifiers is insufficient to prevent large errors appearing. Thus, in using amplifiers for these purposes it is necessary to ensure that the output voltage does not reach the overload level.

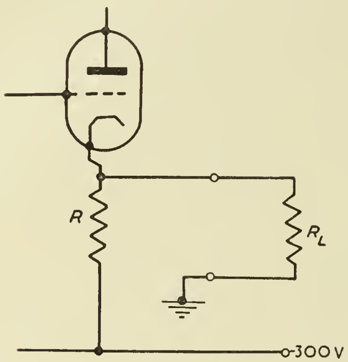


Fig. 58.
Cathode Follower with Load

Besides this limitation on output voltage there is another aspect of the limitation of output from the amplifier. Fig. 58 represents a cathode-follower output stage in which the output voltage to the load R_L is zero when there is no input to the amplifier and the cathode current in this condition is, say, I_0, so that $RI_0 = 300$. Suppose that the cathode current in the valve can increase to $(I_0 + \Delta I)$ and fall to $(I_0 - \Delta I)$ without serious non-linearity. At the higher value there will be a current I_L, say, in R_L, and the current in R will be increased from I_0 to $(I_0 + I_R)$. If the voltage across the load is V_L, the currents will be related by the equations

$$\begin{aligned} V_L &= I_L R_L \\ V_L + 300 &= (I_R + I_0)R \\ \text{or } V_L &= I_R R \text{ since } RI_0 = 300. \end{aligned}$$

Hence
$$I_L R_L = I_R R.$$

Also,
$$\Delta I = I_L + I_R,$$

from which, by elimination of I_R,

$$I_L = \frac{R}{R + R_L} \Delta I$$

This shows that the maximum current which may be drawn by the load is a fraction $R/(R+R_L)$ of the permissible swing of the cathode current from the mean value. Suppose $I_0=20$ mA., $\Delta I=7{\cdot}5$ mA., and $R_L=10{,}000$ ohms; then $R=15{,}000$ ohms, $I_L=4{\cdot}5$ mA., and $V_L=45$ volts. Thus, if a low-current output stage is used to feed a low-resistance load care should be taken that excessive currents are not drawn.

6.5 INPUT AND OUTPUT IMPEDANCES

The requirement that the ideal amplifier should have infinite input impedance and zero output impedance arises from the desirability of being able to feed the output voltage from one amplifier to the input terminals of a number of other amplifiers in parallel without disturbing the value of the output voltage. If the output impedance of an amplifier is Z_O and the open-circuit voltage is V, and if the input impedance of each of the following amplifiers is Z_i, the voltage appearing at the output terminal of the first amplifier when it is feeding n amplifiers in parallel is

$$\frac{V\dfrac{Z_i}{n}}{Z_O+\dfrac{Z_i}{n}}$$

Assuming that Z_O is small compared with Z_i there will be a fractional error of approximately nZ_O/Z_i, and if this is to be, say, one part in a thousand, Z_O must be less than $Z_i/1{,}000n$. Now the input impedance of a high-gain amplifier with a feedback resistor but no input resistor is shown by equation (28) to be equal to $R_F/(1+M)$, and if $R_F=2$ megohms and $M=30{,}000$ the input impedance is roughly 70 ohms. Assuming that the minimum value of input resistor is about 100,000 ohms the input impedance of a summing or sign-reversing amplifier is therefore equal to the resistance itself, to within less than 0·1%.

The normal range of input resistor values is from about 100,000 ohms to a few megohms, so that if ten amplifiers are to be fed in parallel the minimum value of Z_i/n is about 10,000 ohms, and the value of Z_O for a "loading" error of less than 0·1% must be less than ten ohms. The output impedance of a high-gain d.c. amplifier without feedback is the output impedance of the last stage, which is commonly a cathode follower, with an impedance of 500 ohms or less. If the value of M is, say, 30,000, and the value of G is 10, the negative feedback, while reducing the gain by 3,000 times will also reduce the output impedance in a similar ratio, so that the output impedance of the amplifier with feedback is not greater than about 0·2 ohm. Hence for the range of values considered, no appreciable error will result from feeding a number of amplifiers in parallel from one other amplifier.

If the high-gain amplifier has no cathode-follower output stage the output impedance of a normal amplifier stage, without feedback, might be between 10,000 and 50,000 ohms, so that with a feedback reduction of 3,000 times the impedance would be between about three ohms and twenty ohms, which might be acceptable for some purposes.

For single-stage amplifiers, as shown in fig. 46, with $M = 50$, say, the output impedance would probably lie between 10,000 and 50,000 ohms, but the gain with feedback would probably be kept to some lower value than 10 in order to maintain a useful degree of feedback. Similarly, the range of input and feedback resistor values would be kept smaller. For an overall gain of five times the output impedance would be 1,000 to 5,000 ohms, and the input impedance of the amplifier with no input resistor would be 20,000 ohms for a one-megohm feedback resistance. Such impedances can only be ignored for calculations where errors of up to 10%, say, can be ignored. Compensation is possible in certain respects; for example the input resistor to an amplifier may be reduced by an amount equal to the output impedance of the preceding amplifier; but such arrangements are inflexible and of limited practical use. Apart from the inconvenience of using resistor values which bear no simple relations to the gains desired, the compensation depends on the value of M remaining constant. The amplifier of fig. 47 has an output impedance of a few ohms with normal values of G, and this usually acceptable.

In the discussions on amplifiers with feedback it has been assumed that the input and feedback resistors always have exactly the correct values. In practice, of course, the values will not be absolutely accurate even initially, and the errors will most likely change with time. If errors due to inaccurate resistance values are to be less than 0·1% it might seem that at first sight that the values themselves must be within this limit, but it is useful to remember that the basic requirement is that the ratios of the resistor values should be correct, the absolute values being less important. Thus it would be permissible to use a set of resistors which were all, say, exactly 10% above their nominal values. For errors in resistance values to be less than 0·1% the resistors should be wirewound, as carbon resistors cannot be relied on to maintain their values to such close limits, even though the initial values might be correct. In cases where larger errors can be tolerated it may be possible to use high-stability carbon resistors, but it will be advisable to check their values from time to time.

6.6 IMPERFECT INTEGRATORS

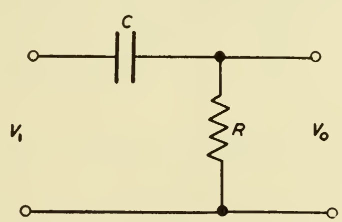
When the amplifier is used as an integrator, with a capacitor as the feedback element, the value of M again has an important influence on the accuracy, though the errors arise in different ways. Assuming that the capacitor is discharged at $t=0$, it follows from equation (6) that, if $T=R_1C$,

$$V_O = -\frac{MV_1}{1+p(T+MT)} \tag{34}$$

which approaches the desired value $-V_1/pT$ as M approaches infinity. The equation may be re-written in the form

$$V_O = -V_1\left(\frac{M}{M+1}\right)\left(\frac{1}{pT}\right)\frac{p(M+1)T}{1+p(M+1)T} \tag{35}$$

which shows that the error due to using a finite value of M appears in two parts. There is a simple "scaling" error represented by $M/(M+1)$; and the pure integration which would be provided by



$$\frac{V_0}{V_1} = \frac{pRC}{1+pRC}$$

FIG. 59

an ideal integrator is, in effect, "contaminated" by the addition of a simple RC differentiation circuit (fig. 59) of time constant $(M+1)T$. The scaling factor is usually negligible, but can be allowed for if desired, so that effectively the error in integration is solely due to the differentiation effect. If, in the integrator, $T=1$ and $M=50{,}000$, then the time constant of the differentiation is $(M+1)$ seconds, which would require values in fig. 59, of say, 50,000 μF and 1·0 megohm. It is sometimes useful to assess the error in a practical integrator by considering the effect of such a differentiating circuit on the proposed input voltage.

Alternatively, the errors may be estimated by finding the output which the practical integrator will give when the input voltage is of some simple form. Suppose, for example, that V_1 is a step function. The equation (34) shows that the integrator behaves like a simple RC "smoothing" circuit (fig. 45) of time constant $(T+MT)$, followed by an amplifier of gain $-M$, so the response to the step is (Ref. 29)

$$V_O = -V_1 M[1-\exp\{-t/(T+MT)\}].$$

This result can also be obtained using equation (35), and it is instructive to derive the solution of this equation in two steps. First, the input step function is operated on by the differentiator term $\dfrac{(M+1)pT}{1+(M+1)pT}$, which gives a function very similar to the original step except that instead of remaining at a constant value after the initial rise the voltage very slowly decays, with time constant $(M+1)T$, in proportion to $\exp\{-t/(M+1)T\}$. There is also a very small change of amplitude corresponding to the term $M/(M+1)$. The second step is the exact integration of this decaying wave, according to the term $1/pT$, which gives

$$\begin{aligned} V_O &= -V_1\left(\frac{M}{M+1}\right)\frac{1}{T}\int_0^t \exp\{-t/(M+1)T\}\,dt \\ &= -V_1 M[1-\exp\{-t/(M+1)T\}] \end{aligned}$$

since $V_O = 0$ when $t = 0$.

Expanding the exponential, and neglecting terms after the third,

$$\begin{aligned} V_O &= -V_1 M\left\{1-\left(1-\frac{t}{T+MT}+\frac{t^2}{2(T+MT)^2}\right)\right\} \\ &= -V_1 M\left\{\frac{t}{T+MT}-\frac{t^2}{2(T+MT)^2}\right\} \qquad (36) \end{aligned}$$

The effect of the t^2 term is to cause the output voltage to rise somewhat more slowly than it would in an ideal integrator. This corresponds to the slow decay of the differentiated step given by the term $\dfrac{(M+1)pT}{1+(M+1)pT}$ in equation (35).

If the error in equation (36) is to be less than 0·1%, the term in t^2 must be less than the term in t in the ratio 10^3 or greater, i.e.,

$$\frac{t^2}{2(T+MT)^2}\cdot\frac{T+MT}{t}<10^{-3}$$

or

$$M>\frac{500t}{T}.$$

Values of M between 30,000 and 100,000 have been mentioned earlier, and if a value of 50,000 is chosen this leads to the restriction $t<100T$ if the 0·1% error limit of error is not to be exceeded. It is not easy to generalize on the importance of this restriction, because although it is quite possible that the total time for some computations will exceed 100 times the shortest integrator time constant in the system, it is unlikely that there will be a steady input to the integrator for the whole of this time. If there is a steady input, then neglecting for the moment the error due to the finite value of M,

$$\begin{aligned} V_O &= -\frac{1}{T}\int_0^t V_1 dt \\ &= -\frac{V_1 t}{T} \quad \text{if } V_1 \text{ is a step function;} \end{aligned}$$

and if $t=100T$, $V_O=-100V_1$, so that the amplifier is likely to overload unless V_1 is less than about one volt.

In any simulator which includes integrator time constants which are short compared with the computing time for one solution care must be taken to see that integration errors of this type are not overlooked. A convenient general rule can be derived from equation (36). Assuming that M is large,

$$\begin{aligned} V_O &= -V_1\left(\frac{t}{T}-\frac{t^2}{2MT^2}\right) \text{ very nearly} \\ &= -V_1\frac{t}{T}(1+\epsilon) \end{aligned}$$

where $\epsilon=-t/2MT$ is the fractional error. Now ϵ is small, so that

$$\begin{aligned} V_O &= -V_1\frac{t}{T} \text{ approximately} \\ &= V_1(2M\epsilon) \end{aligned}$$

and hence, $$\epsilon = \frac{V_O}{V_1}\frac{1}{2M} \text{ approximately.}$$

Thus for a step function input, and for a given value of M, the ratios t/T or V_O/V_1 for a given fractional error can be determined.

Another simple input function which can be used to determine the effect of a finite M value is a square wave. Suppose the input is zero at $t=0$, but rises instantaneously to a value V_1 and remains there for an interval t_1, after which it reverses its sign instantaneously after each successive interval of length t_1. During the first interval the output voltage at time t will be

$$V_O = -V_1\frac{t}{T}\left(1-\frac{t}{2MT}\right),$$

from equation (36), assuming that M is very large; and at the end of the first interval the output voltage will be

$$V_{O1} = -V_1\frac{t_1}{T}\left(1-\frac{t_1}{2MT}\right)$$

so that the error voltage will be $V_1 t_1^2/2MT^2$. Now the reversal at the end of the first interval may be imagined to take place in two stages. First, the input is reduced to zero, and the output voltage remains constant at its value immediately prior to $t=t_1$, which is V_{O1}; the input is then immediately changed to $-V_1$. The total output during the second interval is therefore the sum of V_{O1}, which represents the charge on the capacitor at $t=t_1$, and the voltage due to the integration of $-V_1$ during the elapsed part of the second interval. At the end of the second interval the output voltage is therefore

$$\begin{aligned} V_{O2} &= V_{O1}+V_1\frac{t_1}{T}\left(1-\frac{t_1}{2MT}\right) \\ &= 0 \end{aligned}$$

where both the main terms and the error terms have cancelled. In succeeding pairs of intervals this process is repeated, so that the error never exceeds the value it reaches at the end of the first interval. Even this error is somewhat artificial, since its sign depends on the choice of the sign of V_1 for the first interval. If the first interval were half the length of the succeeding intervals the error would oscillate between two values of opposite sign and of magnitude equal to $V_1 t_1^2/8MT^2$, which is a quarter of the value of the error in V_{O1}.

Thus, the greatest error which can occur in the integration of a square-wave input is equal to the integration error which accumulates during half a period of the wave, and the rule derived for integration of a step function can be applied. Taking $M = 50{,}000$ and permissible error as $0{\cdot}1\%$, this means that integration of square waves will be satisfactorily accurate provided the periodic time of the wave is not greater than about 200 times the integrator time constant.

If the input to the integrator is a sine wave of frequency $\omega/2\pi$,

$$V_O = -\frac{MV_1}{1 + j\omega(T + MT)}$$

where the p of equation (34) has been replaced by $j\omega$.

The modulus of the ratio $\dfrac{V_O}{V_1}$ is

$$\left|\frac{V_O}{V_1}\right| = \frac{1}{\sqrt{\dfrac{1}{M^2} + \omega^2T^2\left(1 + \dfrac{1}{M^2}\right)}} \tag{37}$$

whereas in an ideal integrator with $M \to \infty$ the modulus would be $1/\omega T$. Re-writing equation (37),

$$\left|\frac{V_O}{V_1}\right| = \frac{1}{\omega T\sqrt{\left(1 + \dfrac{1}{M^2}\right)\left\{1 + \dfrac{1}{\omega^2T^2(1 + M^2)}\right\}}}$$

$$\doteqdot \frac{1}{\omega T}\left(1 - \frac{1}{2M^2}\right)\left\{1 - \frac{1}{2\omega^2T^2(1 + M^2)}\right\}$$

taking binominal expansions to two terms.

The error due to the $1/2M^2$ term will nearly always be negligible, so that for $0{\cdot}1\%$ error

$$2\omega^2T^2(1 + M^2) = 1{,}000, \text{ or, if } M \text{ is } 50{,}000,$$

$$\omega T \doteqdot 4{\cdot}5 \times 10^{-4}, \text{ or } \frac{1}{f} \doteqdot 1{\cdot}4 \times 10^4 T;$$

so that provided the period of the sine wave is less than about 10,000 times the integration time constant the error will be less than $0{\cdot}1\%$.

For a sine wave input there will be, besides the small error in amplitude, an error of phase between V_O and V_1, in addition to the normal 90° lag which would be given by an ideal integrator. The magnitude of the error is equal to

$$\tan^{-1}\{1/\omega(T + MT)\} \text{ radian.}$$

If $M = 50{,}000$ and the period of the wave is $1{,}000T$, this gives about $0{\cdot}2$ degree, which is not usually important. This error decreases as the periodic time of the wave decreases relative to the time constant T.

In an integrator with sinusoidal input the ratio of output to input voltages is less sensitive to changes in the value of M than in the case of a summing or sign-reversing amplifier. This can be shown by differentiating equations (37) and (31) with respect to M and comparing the derivatives. This effect is a consequence of the phase-shift in the feedback network, and was treated by H. S. Black in his classic paper on feedback amplifiers (Ref. 13).

As in the case of amplifiers with resistive feedback the integrator can perform accurately only if the values of the feedback and input impedances are accurate, though it is the time constant given by the product of resistance and capacitance rather than their separate values which must be accurate. There is, however, an additional

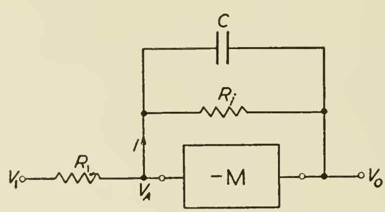


Fig. 60. Integrator with Leaky Capacitor

source of error in the integrator because the capacitor is not free of resistive effects, as an ideal capacitor would be. In particular the imperfect insulation of the dielectric gives the effect of resistance in parallel with the capacitance, as shown by R_i in fig. 60. Taking R_i into account, the dependence of V_O on V_1 can be derived in the usual way:

$$V_O = -MV_A$$

$$I = \frac{(V_A - V_O)}{R_i} + (V_A - V_O)pC$$

$$I = \frac{(V_1 - V_A)}{R_1},$$

and after eliminating I and V_A,

$$V_O\left(\frac{1}{MR_1} + \frac{pC}{M} + \frac{1}{R_iM} + pC + \frac{1}{R_i}\right) = -\frac{V_1}{R_1}$$

For the present purpose it is permissible to neglect pC/M in comparison with pC, and $1/R_iM$ in comparison with $1/R_i$, so that

$$V_O\left(\frac{1}{MR_1}+pC+\frac{1}{R_i}\right) = -\frac{V_1}{R_1}$$

$$\text{or, } V_O = -\frac{V_1}{pT+\dfrac{1}{M}+\dfrac{R_1}{R_i}}$$

In an ideal integrator the denominator would contain pT only, and it appears from this equation that in addition to the error due to the non-infinite value of M, which has already been discussed, there is now an error due to the non-infinite value of R_i. The magnitude of this error can be illustrated by replacing the arrangement of fig. 60 by an integrator using a perfect capacitor and an amplifier of gain M'. For this arrangement, from equation (34),

$$V_O = -\frac{V_1}{pT+\dfrac{1}{M'}}$$

and this will give the same performance as the arrangement of fig. 60, if

$$\frac{1}{M'} = \frac{1}{M}+\frac{R_1}{R_i}$$

or

$$M' = M\frac{1}{1+\dfrac{MR_1}{R_i}}$$

If $M=50{,}000$ and $R_1=2\times10^6$ ohms, R_i must be at least 10^{12} ohms if M' is to be within 10% of M, or 10^{11} ohms if $M'=M/2$. Such insulation resistances can be achieved in capacitors of 1·0 μF., which is the order of size commonly used in computing machines of the type discussed in earlier chapters, if polystyrene is used as the dielectric. The "insulance" of polystyrene, i.e. the product of insulation resistance and capacitance, is approximately 200,000 megohm-microfarad, which is some ten to twenty times better than mica, and the dielectric absorption effects are smaller in about the same ratio.

Resistive losses in the dielectric of the capacitor, corresponding to a non-zero power factor, may be represented by adding a resistor R_s in series with the capacitor of fig. 4*a*. Then;

$$\frac{V_O}{V_1} = -\frac{R_s + \dfrac{1}{pC}}{R_1}$$

$$= -\frac{1}{pCR_1} - \frac{R_s}{R_1}$$

Thus in addition to the pure integral term the output also contains a term proportional to the input voltage. The power factor of polystyrene capacitors as used in analogue computers is about 0·005, which means that R_s is usually negligible.

6.7 THE DIFFERENTIATOR

Besides its use as summing amplifier, sign-reverser and integrator, the high-gain d.c. amplifier may also be used, with capacitor input and resistor feedback, as a differentiator (fig. 44). In ideal circumstances the differentiator is as satisfactory as the integrator, but in practice differentiation is usually avoided whenever possible. There are three reasons for this. First, the differentiator arrangement is more difficult to stabilize, especially in a flexible machine where it may be required to use the same amplifier as a summing amplifier on one occasion, and as integrator or differentiator on other occasions. Second, the differentiating action tends to increase the "spikiness" of a varying input voltage, and hence tends to increase the chance of overload. Third, and probably most important, the differentiator tends to degrade the "signal-to-noise ratio" of the output voltage by amplifying the residual power-supply ripple and other higher-frequency disturbances which are always present to some extent. The question of stability has already been discussed (Section 5.2), and the other objections can be illustrated more fully by a simple analysis of the arrangement of fig. 44. Following the usual procedure it is easily shown that

$$V_O = -V_1\left(\frac{pT}{1 + \dfrac{1}{M} + \dfrac{pT}{M}}\right)$$

which may be re-written

$$V_O = -\left(\frac{M}{M+1}\right)\left(\frac{V_1 pT}{1+\frac{pT}{M+1}}\right) \tag{38}$$

This equation shows that, compared with an ideal differentiator, the use of a finite value for M gives an error which appears in two parts. There is first a "scaling" error represented by $M/(M+1)$, and secondly, the pure differentiation is, in effect, "contaminated" by the addition of a simple RC smoothing circuit (fig. 45) of time constant $T/(M+1)$. The error due to this contamination corresponds closely with the error due to using a finite value of M in an integrator.

If the input to an ideal differentiator is a step function of voltage the output is an impulse function, having an infinite amplitude and infinitesimal duration such that the "area" of the impulse, i.e. the product of amplitude and duration, is proportional to the amplitude of the input step. If a step function is applied to a practical differentiator the output rises to a value given by setting $p \to \infty$ in equation (38), which is

$$V_O = -MV_1,$$

assuming that the amplifier does not overload. In practice, of course, overloading would occur for almost all values of M and V_1 of practical interest, so that the differentiator is useless when its input contains step functions. Even when the input rises at a finite, rather than an infinite rate, overloading may occur. If the input voltage rises at a steady rate of k volts per second, then $V_1 p$ in equation (38) may be replaced by k, and the output voltage is, ignoring the $pT/(1+M)$ term for the moment,

$$V_O = -\frac{M}{M+1}(kT) = -kT \text{ very nearly,}$$

from which some estimate of the permissible rate of change of the input voltage can be made when T is known.

If the input voltage to a differentiator is sinusoidal, of frequency $\omega/2\pi$, then equation (38) becomes

$$\frac{V_O}{V_1} = -\left(\frac{M}{M+1}\right)\left(\frac{j\omega T}{1+\frac{j\omega T}{M+1}}\right) \tag{39}$$

so that, at frequencies such that $\omega T \ll M$, the differentiator behaves as an amplifier with a gain which increases linearly with frequency.

Now suppose that a differentiator has a time constant of 1·0 second, and the input has a small 50 c/s content of magnitude v. Then the 50 c/s content of the output voltage will be very nearly equal to $\omega v = 314v$, which will be intolerable unless v is a small fraction of a volt. If chopper-stabilized amplifiers are being used the input signal may contain, say, 4 mV of ripple at 400 c/s which would give an output ripple of about ten volts. At still higher frequencies the gain, according to equation (39) will continue to rise towards the asymptotic value M, but as the frequency rises above a few hundred cycles per second it is likely that the value of M will fall, so that the maximum value will be appreciably less than the low-frequency value of, say, 50,000. For an amplifier of the type shown in fig. 52 the maximum gain of a differentiator with $T = 1$ would be several thousand times.

Thus, a single differentiator with a time constant of the order of 1·0 second is liable to give a serious increase in the noise content of the variable voltage, and the use of two or more such differentiators in any tandem connection is usually out of the question.

The liability of a differentiator to overload when the input voltage varies rapidly, and the error due to the $pT/(M+1)$ term in the denominator of equation (38) are both aggravated if T is increased, so in cases where the use of a differentiator is unavoidable it is often advisable to use as small a time constant as possible. A smaller value of T reduces the amplification of residual ripple and noise, but the amplitude of the desired voltage is reduced in the same ratio, so that the signal-to-noise ratio is apparently not improved. However, this argument assumes that the ripple is fed into the differentiator with the legitimate input voltage, whereas there may be appreciable injection of ripple within the differentiator amplifier, say from the heater of the first valve, or from the chopper amplifier. In such cases it will be advantageous to use a small value of T and compensate for the reduction of output voltage by raising the level of the input voltage. A reduction of T with a compensating increase of gain *after* the differentiator would generally leave the signal-to-noise ratio unchanged.

7

NON-LINEAR COMPUTING ELEMENTS

It has already been shown that, except for the simplest simulators, computing elements capable of performing other operations than adding, integrating and differentiating are needed. The most important of these additional elements are multipliers, dividers, and generators of trigonometric and other functions. Limiting and "backlash" devices, and trigger circuits are also used. A large variety of methods have been proposed to perform these functions, some of them wholly electronic, and some electro-mechanical. Although in the simulators described elsewhere in this work it is generally necessary to use electronic devices because the time lags in mechanical devices are not acceptable it is often easier to achieve high precision by mechanical means, and electro-mechanical computing elements are sometimes used in positions where the rates of variation are sufficiently slow.

7.1 VARIABLE MARK/SPACE MULTIPLIER

Several different principles have been employed in electronic multipliers, and among these is the use of a square wave in which the "mark/space" ratio is varied in accordance with one input voltage and the amplitude of the wave is made proportional to the other

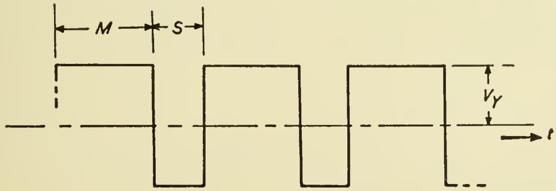


FIG. 61. Square-Topped Wave with "Mark" longer than "Space"

voltage. Such a wave is shown in fig. 61, where the "mark" and "space" intervals are respectively M and S seconds long. If the input voltages are V_X and V_Y, the M/S ratio is adjusted so that

$$V_X = k_1\frac{M-S}{M+S} \quad \text{or} \quad \frac{k_1+V_X}{k_1-V_X} = \frac{M}{S} \tag{40}$$

where k_1 is a constant. The wave has positive and negative values equal to $+V_Y$ and $-V_Y$, so that during one complete cycle the area under the curve is MV_Y while the voltage is positive, and SV_Y while it is negative. The difference in these two areas is proportional to the mean value, or d.c. component of the wave, so that if the wave is passed through a filter to remove the alternating components the output will be

$$V_O = k_2(MV_Y - SV_Y)$$

where k_2 is a constant. Thus, from this equation and equation (40)

$$V_O = k_2V_Y(M-S) = k_2V_Y\frac{V_X(M+S)}{k_1} = V_XV_Y\frac{k_2}{k_1}(M+S).$$

$(M+S)$ is the periodic time for one cycle of the wave, and assuming this is constant, $k_2(M+S)/k_1$ is also constant, equal to K, say, so that

$$V_O = KV_XV_Y$$

An important feature of this principle is that it can accept both positive and negative inputs for both V_X and V_Y, and gives an output voltage of the correct sign. Thus, as V_X decreases from the

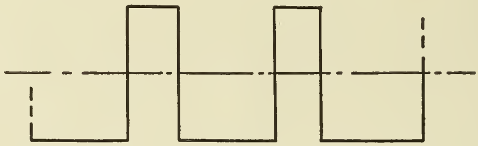


FIG. 62

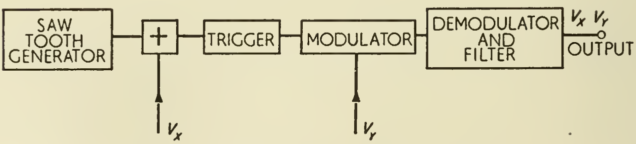


FIG. 63. Variable Mark/Space Multiplier

value indicated in fig. 61 the mean value of the wave will fall, until when $V_X = 0$ the mark and space are equal and the mean value is zero. If V_X becomes negative, $M < S$, so the mean value is negative. Provided a suitable modulator is used, the effect of reversing the sign of V_Y is to replace the wave of fig. 61 by fig. 62, which obviously has a mean value of the same magnitude but opposite sign.

A multiplier using the variable-mark-space principle was designed by H. SUTCLIFFE about 1947. Another multiplier using the same principle with a different circuit was developed by the author and D. W. ALLEN (Ref. 30) and later improved by M. SQUIRES, and the operation of this device is illustrated in fig. 63. A saw-tooth generator produces a voltage with the wave-form shown in fig. 64*a*. The exact shape of this wave is unimportant provided that the rise is linear and that the return is either very rapid or else linear. The wave is adjusted to be symmetrical about earth potential, so that the instants at which the voltage passes through zero are all equally spaced in time. The V_X voltage is then added to the saw-tooth wave, and the sum is fed into a trigger device which changes very rapidly from one state to another when the input voltage passes through zero. When the trigger is in one state it gives a large positive output voltage, and in the other state it gives a large negative voltage. Assuming for the moment that V_X is zero, the trigger will spend equal intervals alternately in its two states, and will give an output wave of the form shown in fig. 64*b*.

Suppose now that V_X is given a constant positive value. The combined voltage will now vary with time as shown in fig. 64*c*, and the output from the trigger will be as in fig. 64*d*, with a mark/space ratio greater than unity. It is easily shown in fact, that provided the sides of the saw-tooth wave are straight the mark/space ratio satisfies equation (40). The output from the trigger is passed to a modulator which has as a second input the voltage V_Y. The action of the modulator, which will be described more fully later, is to produce a wave of the same mark/space ratio as (*d*), but with a peak-to-peak

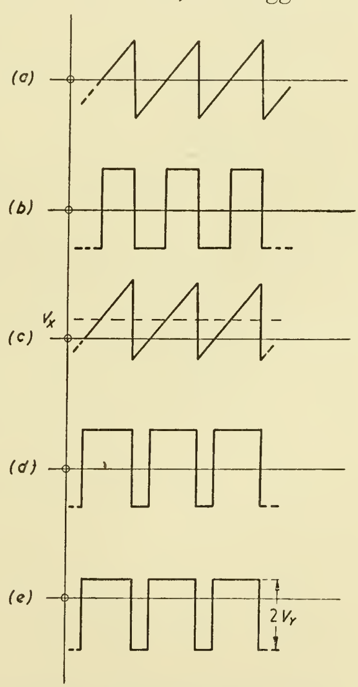


FIG. 64. Multiplier Waveforms

amplitude equal to $2V_Y$, as in (*e*). This wave bears some resemblance to that of fig. 61, but there is the important difference that whereas fig. 61 shows a wave with equal positive and negative peak values and a positive mean value, the wave of fig. 64*e* has unequal peak values and zero mean value. The output voltage from the modulator is assumed to pass through a capacitor, so it must always have a zero mean value, and the positive and negative peak values vary with V_X, as well as with V_Y, though the peak-to-peak value is independent of V_X. This output voltage is fed into a demodulator and filter, described more fully below, which gives an output proportional to $V_X V_Y$.

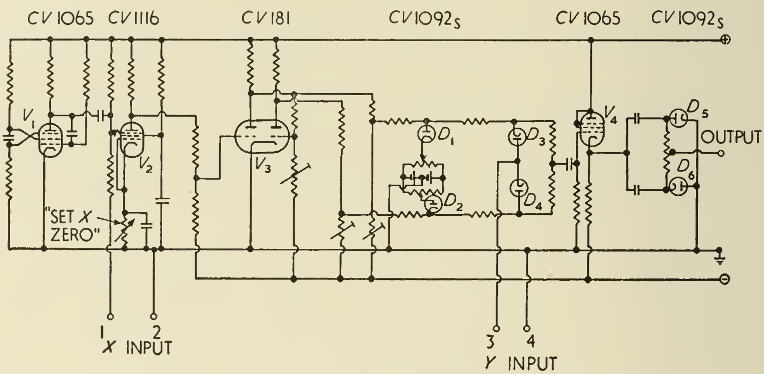


FIG. 65. Variable Mark/Space Multiplier

Fig. 65 shows the circuit arrangement of an early form of the "variable mark/space" multiplier. V_1 is a transitron oscillator which generates the saw-tooth wave at 1,000 c/s. V_2 serves as the trigger of fig. 63, although it is not strictly a trigger valve, but is so arranged that the anode-current is large when the suppressor grid is positive and very small when the suppressor is negative. V_3 is a double-triode phase-splitter which, with diodes D_1, D_2, D_3, D_4, forms the V_Y-modulator. V_4 is a cathode-follower buffer stage, and diodes D_5, D_6, are demodulators. The action of the modulator can be followed with the help of the simplified circuit of fig. 66. The terminal A receives the output from one side of the phase splitter, which is a wave having a mark/space ratio corresponding to V_X and a constant amplitude which is larger than the maximum value of V_Y. Terminal B receives the corresponding wave of opposite phase. When A is positive B is negative, so that diodes D_1 and D_2 conduct, and assuming that their impedances in the conducting condition are zero the two points C and D are at earth potential. If V_Y is positive D_3 conducts and D_4 does not, so that E is at potential V_Y, and assuming the resistors have equal values, the output terminal G has a potential

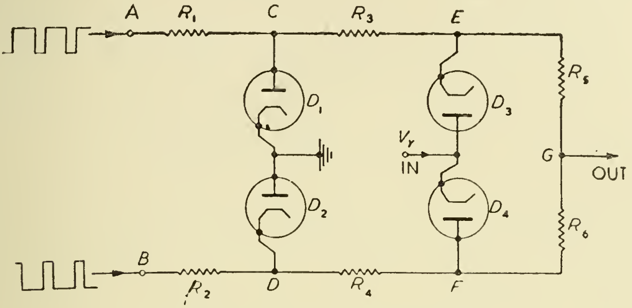


FIG. 66. Modulator for Multiplier

$2V_Y/3$. When A becomes negative and B positive the diodes D_1 D_2 no longer conduct, but D_3 D_4 conduct so that all the points $E, F, G,$ are at potential V_Y. Thus the voltage at G alternates, with the correct mark/space ratio, between the values V_Y and $2V_Y/3$, which is equivalent to a square wave of peak-to-peak amplitude $V_Y/3$ plus a steady voltage of value $\frac{1}{2}(V_Y+2V_Y/3)=5V_Y/6$. If V_Y is negative the output is similar except that it is reversed in sign.

This modulator has the advantages that the amplitude of the wave from the phase splitter is unimportant provided it is large enough, and it operates with a single V_Y input, whereas some modulators require a phase splitter to provide both V_Y and $-V_Y$. The chief disadvantages are that the output contains a d.c. component, and careful matching of diodes is necessary for best results, especially if V_Y is small. The battery and adjustable resistors between D_1 and D_2 in fig. 65 are provided to improve the performance of the diodes.

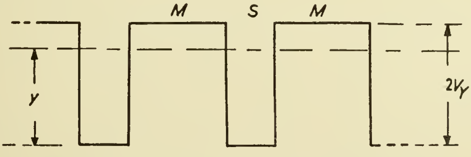


FIG. 67. Input Wave to Demodulator

The demodulator is fed with a wave of the form shown in fig. 67, which has the correct amplitude and mark/space ratio, but has zero mean value. Thus, if the peak-to-peak amplitude is $2V_Y$, and if the negative and positive peak amplitudes are respectively y and $(2V_Y - y)$, then

$$yS = M(2V_Y - y),$$

or

$$y = 2MV_Y/(M+S),$$

and

$$2V_Y - y = 2SV_Y/(M+S).$$

Now D_5, D_6, (fig. 65) and the associated components comprise, effectively, two peak rectifiers, and if the time constants are long compared with the periodic time of the square wave the two capacitors will charge, respectively, to $+2MV_Y/(M+S)$ and $-2SV_Y/(M+S)$. The voltage at the centre tap of the resistor is the mean of these two values, which is $V_Y(M-S)/(M+S) = V_X V_Y/k_1$ by equation (40). The residue of alternating voltage at the output terminal is negligible in many circumstances, but can be reduced by further filtering if desired.

With careful adjustment this multiplier will give results which are accurate to within about 2% of the maximum output voltage. The errors are largest when the output voltage is near zero, because the diode impedances then have an important influence. This multiplier has a rather poor frequency response, chiefly on account of the filter associated with the demodulator. This filter is rather crude, and the saw-tooth frequency is only 1,000 c/s, so that appreciable phase-shifts occur if the input voltages have components at more than one or two cycles per second.

An improved version of the variable mark-space multiplier is shown in fig. 68. The saw-tooth, with a frequency of 2,500 c/s is generated by a separate generator which can be common to a number of multipliers. The combined saw-tooth and X voltage are not applied directly to the grid of V_2, which is the first valve of a Schmitt trigger circuit (Section 7.8), but through a circuit which includes the double diode V_1. The object of this circuit is to avoid drawing grid current in the grid circuit of V_2. While the combined voltage is negative the circuit has no effect, except that the negligibly small forward impedance of the right-hand diode is in series with the lead to the grid. Then at a point in the voltage rise more positive than the triggering point, but not sufficiently positive for the onset of

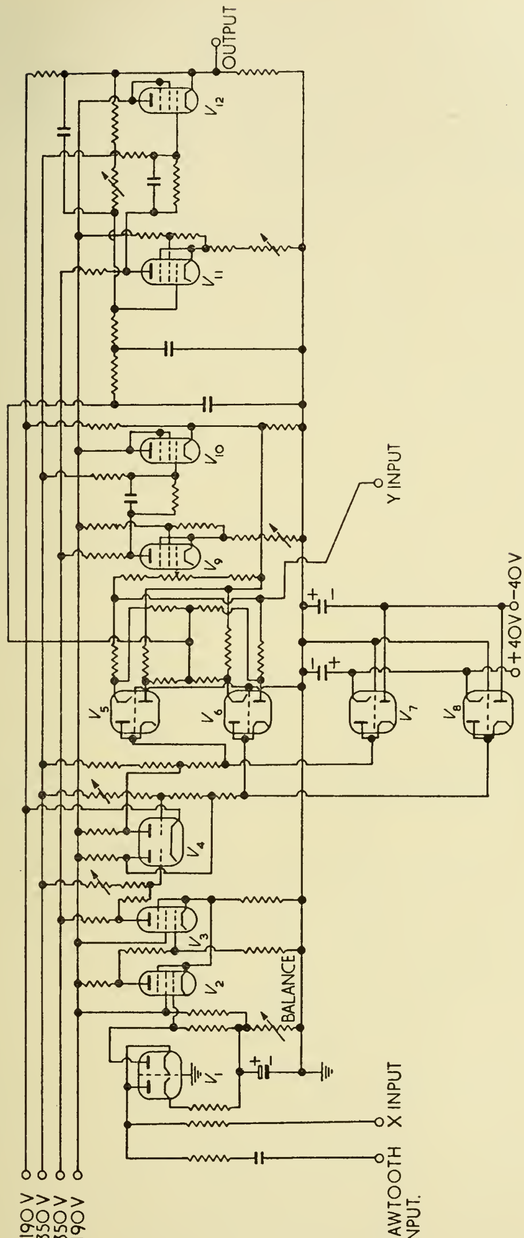


Fig. 68. Improved Multiplier

grid current, the left-hand diode conducts and the right-hand diode cuts off. Thus the grid is not driven further positive, although the combined voltage continues to rise, and the left-hand diode ensures that the impedance presented to the combined voltage remains unchanged and avoids d.c. effects which might be caused by diode currents charging the input capacitor.

The double triode is a phase splitter which feeds the modulator ($V_5 V_6$). This modulator is different from the one shown in fig. 65, and has several advantages. It is less susceptible to differences in curvature of the diode characteristics, and more important, it gives an output voltage corresponding to fig. 61, such that the mean value of the output wave is proportional to $V_X V_Y$, and no demodulating action is necessary. It requires, however, that the square-wave input shall have positive and negative peak values which are accurately limited to equal and constant values; and for any input voltage V_Y an equal and opposite voltage $-V_Y$ must be provided, from a low impedance source. The limiting action is provided by the diodes V_7 and V_8, and the $-V_Y$ signal is provided by the reversing amplifier V_9 and cathode follower V_{10}. The modulator is shown also

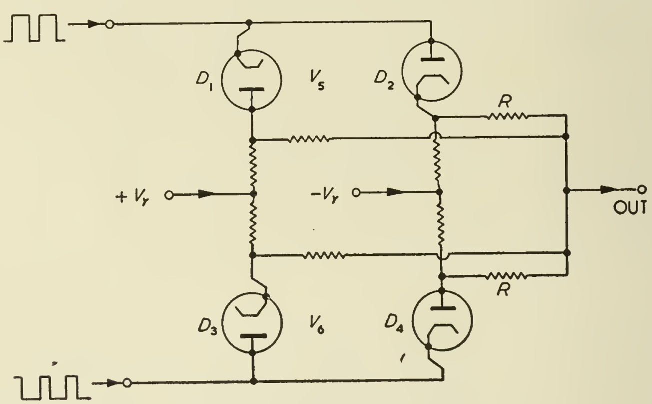


Fig. 69. Modulator Circuit

in fig. 69. Assuming that the amplitude of the square wave is greater than V_Y or $-V_Y$, the diodes D_2 and D_4 conduct during the positive part of the wave, and D_1 and D_3 conduct during the negative part. When D_2 and D_4 conduct the two resistors R form a centre-tapped pair between equal positive and negative voltages and, assuming symmetry, there is no potential at the output due to $-V_Y$. However, since D_1 and D_3 are not conducting $+V_Y$ is connected to the output terminal by two series pairs of resistors in parallel (fig. 70). Thus, the output voltage is a proportion of V_Y, say qV_Y, where q is a constant determined by the values of the resistors. During the negative part of the square wave, diodes D_1 and D_3 conduct, so that the output voltage is $-qV_Y$, where q has the same value as before, provided the resistor values are suitably chosen. The output voltage

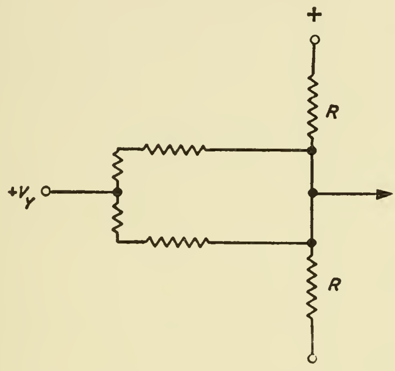


FIG. 70. Resistor Network

therefore has the correct waveform and has peak amplitudes $\pm qV_Y$, so that it is only necessary to remove the alternating component by means of a filter to give a voltage proportional to V_XV_Y. Two stages of filtering are provided by the components in the grid circuit of V_{11}, which is a d.c. amplifier. V_{12} is a cathode follower to give a low output impedance.

This multiplier gives answers which are accurate to better than 1% of full scale, and this accuracy is maintained up to frequencies of 3 or 4 c/s for the V_X input and up to about 20 c/s for the V_Y input. The reason for the difference is that the only appreciable reactances which give errors when the V_Y voltage varies are the capacitors in the output filter, whereas for variations in the V_X voltage the capacitor used in the circuit for adding the saw-tooth and V_X voltages gives attenuation greater than and additional to that given by the filter. Similarly, the phase-shift due to the filter alone is such that for variations of V_Y up to 10 c/s the output phase is in error by not more than 10°, whereas variation in V_X of 4 c/s also give a phase error of 10°. Further development of the variable-space-mark method, with considerable improvement in accuracy, has recently been reported (Ref. 31).

7.2 OTHER MULTIPLIERS

Of the other principles which have been used for analogue multiplication two of the most important are first, the "biased diode" principle, which will be described in Section 7.5, together with other elements using biased diodes, and second, the "crossed field" multiplier (Ref. 32). This latter device makes use of a special cathode-ray tube which has, in place of the usual fluorescent screen, a pair of collector plates so arranged that the undeflected electron beam falls equally on both the plates; but a deflection of the beam in the vertically upward (Y) direction causes an increase of current to the upper collector, and vice versa. Deflection of the beam in the horizontal (X) direction does not change the current distribution. The tube has the usual two pairs of deflector plates, and in addition has a coil which produces a magnetic field in a direction parallel to the undeflected beam. One of the voltages to be multiplied is applied to the horizontal-deflection (X) plates, and the other voltage is used to produce a proportional current in the coil. In the presence of the X-deflecting voltage the electron beam is no longer parallel to the magnetic field, so that there is an additional deflection, this time in the Y direction, due to the magnetic field. This deflection is proportional to the product of the strength of the magnetic field and the X deflection, i.e. to the product of the two input voltages. The currents from the collecting plates are passed to an amplifier which gives an output voltage of such polarity that, when applied to the Y plates of the tube, the beam deflection is reduced, and the gain of this amplifier is made so large that the beam returns almost to its undeflected position. Thus, the Y-deflection voltage is proportional to the Y deflection which would be produced by the two input voltages, i.e. it is proportional to the product of the two input voltages.

This multiplier has been developed into a satisfactory computing element, and in a version designed by J. A. Roberts and D.C. Pressey and engineered by Southern Instruments Ltd. (Camberley), gives results accurate to about 0·5% of full scale at frequencies of several kc/s. This wide frequency range makes this type of multiplier suitable for use in "repetitive" simulators, as described in Section 10.2.

Several other multipliers have been proposed, including: double-modulation systems; a "probability" multiplier depending on the fact that the combined probability of two independent events is the product of their separate probabilities; and a "logarithmic" multiplier in which voltages are generated proportional to the logarithms of the input voltages. The last two methods have the interesting facility that more than two input voltages can be multiplied in one operation, but generally these principles have not been widely applied.

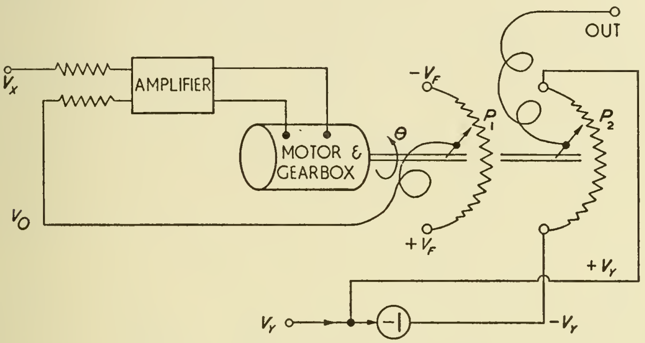


FIG. 71. Electro-Mechanical Multiplier

Besides the purely electronic methods a number of electro-mechanical multipliers have been designed, and fig. 71 shows a popular arrangement. The amplifier, motor, and potentiometer P_1 fed with fixed voltages $\pm V_F$ constitute a straightforward position servo; whenever the input voltage V_X is different from $-V_O$, where V_O is the voltage on the slider of P_1, the net input to the amplifier is not zero, and the motor turns in a direction to reduce the amplifier input voltage to zero. Thus, if the amplifier has a sufficiently high gain the motor turns until $V_O = -V_X$, and if the potentiometer is linear, i.e. if the slider voltage is proportional to the angle of deflection θ, then $\theta = kV_X$, where k is a constant. The angle θ is measured from the centre point of the potentiometer, being taken as positive on one side and negative on the other side of this point. Now if the potentiometer P_2 is also linear and is fed with voltages $\pm V_Y$, where V_Y is the second input voltage, the voltage at the potentiometer slider is proportional to $V_Y\theta$, i.e. to $V_Y kV_X$, or to $V_X V_Y$.

This multiplier has the advantage of simplicity and gives a correct output whatever the signs of V_X and V_Y, but it has a number of disadvantages which limit its application. A fairly powerful amplifier is needed to drive the motor, and even so the frequency response of the system is severely restricted by the limited angular acceleration of the motor shaft. The accuracy of multiplication is largely determined by the accuracy of the potentiometers, and particularly by the differences between them. Considerable departure from linearity can be tolerated provided the two potentiometers are identical, so that for any value of θ the proportion of V_F given by P_1 is the same as the proportion of V_Y given by P_2. In practice, however, the variations from linearity tend to be erratic and with ordinary good-quality potentiometers may be up to 2%. If high-grade cam-corrected potentiometers are used the errors can be reduced to 0·2% or better, but these potentiometers need higher driving torques, and the larger motor tends to degrade the frequency response. If a multiplier of this type is used to feed directly into a resistive load the part of the potentiometer (P_2 in fig. 71) between slider and earth is shunted by the load and there is consequently an error in the voltage applied to the load unless the load resistance is very high compared with the potentiometer. A common method of reducing this error is to insert a buffer amplifier, but a simple alternative is to connect a resistor, of value equal to the load resistance, between the slider of the feedback potentiometer (P_1 fig. 71) and earth. It can easily be shown that this provides almost exact compensation. This assumes that the two potentiometers have equal resistances, but if the resistances should be unequal a similar measure of compensation can still be obtained by connecting to the feedback potentiometer a resistor whose value bears the same relation to this potentiometer as does the load resistance to the output potentiometer. In a similar manner, compensation for the shunting effect of the feedback resistor on the feedback potentiometer can be obtained by connecting a suitable resistor to the output potentiometer.

A multiplier of this type has been designed at R.A.E. and engineered and manufactured by Dobbie McInnes Ltd. (Glasgow). In this version there is one feedback potentiometer and five "ganged" multiplying potentiometers, so that four input voltages can each be multiplied by a fifth input voltage to give four separate product voltages. This multiplier gives errors, for slowly-changing inputs, of less than 0·2%, and it will follow sinusoidal inputs of large amplitude at 0·2 cycle/second with an amplitude error of 0·2%.

Another multiplier of this type, but capable of much more rapid operation has been built by the author and D. W. Allen, using an electromagnetic device designed by A. E. Laws. This device is similar to one type of polarized relay, but instead of closing electrical contacts the current in the coil causes a light shaft, carried in ball bearings, to turn through a small angle. This angle is proportional to the current over a range of $\pm 5°$, and for use in the multiplier an arm about 4″ long was attached, carrying two insulated sliders

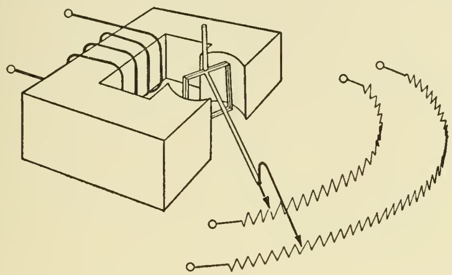


FIG. 72. High-Speed Multiplier

operating on two short straight potentiometer windings. The arrangement is shown diagrammatically in fig. 72, and it replaces the motor and two potentiometers of fig. 71. The circuit diagram of the complete multiplier is shown in fig. 73. This multiplier, again, is not capable of great accuracy because of the difficulty of matching the potentiometers, but it is simple and fairly compact, and has a time of response (to 90% for a step input) of about 10 milliseconds.

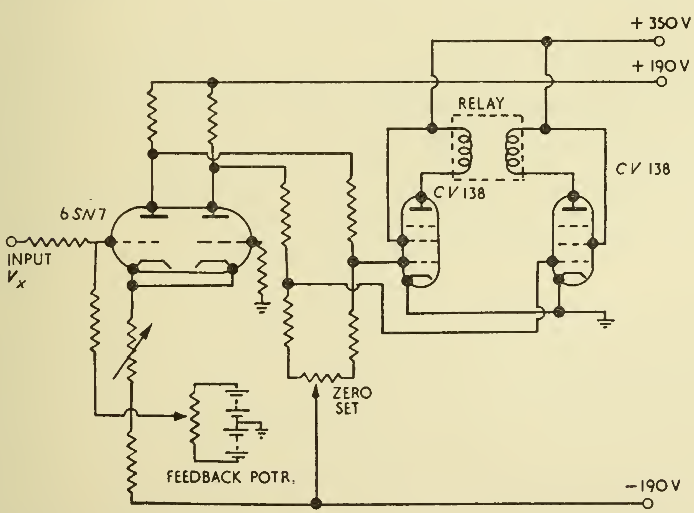


FIG. 73. Circuit Diagram of High Speed Electro-Mechanical Multiplier

7.3 DIVIDERS AND SQUARE-ROOT DEVICES

It is sometimes necessary to use dividers in analogue computing machines, though for reasons which will appear they are avoided whenever possible. An analogue divider is a device which, when fed with two voltages V_X and V_Y gives as output a voltage V_Z such that $V_Z = KV_Y/V_X$. It is not possible to make a divider which will operate correctly when the V_X voltage passes through zero, since $V_X \to 0$ requires $V_Z \to \infty$ if V_Y is finite, and V_Z is limited by the overloading of the amplifiers used. Thus any computer which uses a divider should be so arranged as not to demand division by small numbers, and it is generally desirable to keep the range of divisors as small as possible.

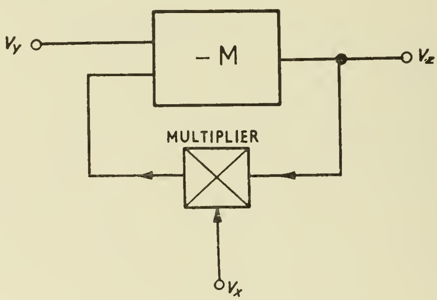


FIG. 74. Feedback-Type Divider

There are several principles on which dividers have been based, including the "reciprocal" method, which uses a biased-diode arrangement described later, the "feedback" method, and the electromechanical method. The feedback method makes use of a multiplier and a high-gain summing amplifier, connected as shown in fig. 74. The relation between the voltages is given by

$$V_Z = -M(V_Y + V_Z V_X)$$

or,

$$-MV_Y = MV_Z V_X + V_Z$$

and provided $MV_X \gg 1$,

$$V_Y = -V_Z V_X, \text{ very nearly,}$$

or

$$V_Z = -V_Y/V_X.$$

This result represents an example of the well-known principle that the overall characteristic of a high-gain feedback amplifier is the inverse of the characteristic of the feedback network. At first sight it might be assumed, apart from the limitation imposed on V_X by overloading when V_X is small, and by the requirement $MV_X \gg 1$, that positive and negative values of both V_X and V_Y could be handled, but this is not so. Assuming that the multiplier itself is able to deal with both signs of V_X and V_Y, if the system is stable for one sign of V_X it will generally be unstable for the other sign. Thus, if the amplifier has, as is usual, an overall phase shift of 180° at low frequencies, stability requires that there should be no phase reversal in the feedback path. In the present case this requires that the output voltage $V_X V_Z$ of the multiplier must be of the same sign as its input V_Z; i.e. V_X must be positive. If it is required to work with negative values of V_X an additional phase reversal must be inserted in the loop, but the divider will then be unstable for positive values of V_X. Besides these elementary considerations of phase reversals, the question of stability also involves consideration of the amplitude and phase characteristics of the multiplier at higher frequencies. The multiplier is, in effect, a variable gain device operating in the feedback loop, and the whole divider will be stable only if the Nyquist Criterion (Refs. 13-16) is satisfied for all likely values of V_X. Provided overloading is avoided there is no restriction on the value or sign of V_Y.

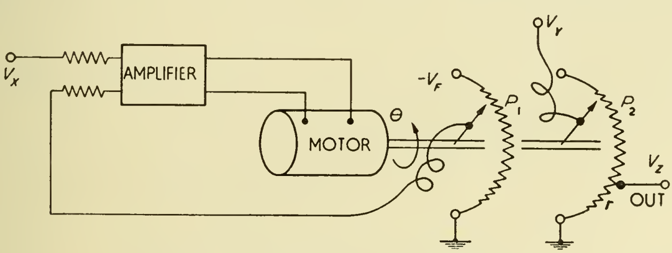


FIG. 75. Electro-Mechanical Divider

The electro-mechanical divider uses an amplifier, motor, and first potentiometer as in the type of multiplier shown in fig. 71, but with a different arrangement for the second potentiometer. The arrangement is shown in fig. 75 and since the divider is only effective for positive values of V_X it is only necessary to consider positive values of θ, so the first potentiometer is connected between $-V_F$ and earth. The second input voltage V_Y is connected to the slider of P_2, which has a total resistance R_0 extending over an angular range θ_0, so that the resistance R between slider and earth is $\theta R_o/\theta_o$. The output is taken from a tap on the potentiometer at a resistance r from the earthy end, and the output voltage V_Z is therefore $rV_Y/R = rV_Y\theta_o/\theta R_o$. Now as explained in connection with the electro-mechanical multiplier $\theta = KV_X$, so that $V_Z = rV_Y\theta_o/R_oKV_X = kV_Y/V_X$, where k is a constant.

This divider has the advantage of simplicity, and the stability problem is no greater than for the multiplier, but there are some disadvantages. In particular, the limitation on the value of V_X is even more severe than that imposed by the amplifier overload. This is because the relation $V_Z = rV_Y/R$ is valid only for values of R greater than r; if R is less than r the relation gives V_Z greater than V_Y, which is absurd. Thus, V_X can only be allowed to vary between some maximum value, say V_M, which corresponds to $\theta = \theta_o$, and a lower limit rV_M/R_o, and although this lower limit can be reduced by reducing r this gives correspondingly lower output voltages. Furthermore, if r is made a small fraction of the whole winding it will include relatively few turns of wire, and the accuracy of the computation may be degraded if the jump from one turn to the next is greater than errors due to other causes. The output voltage can never be greater than V_Y, but there is no restriction on the value or sign of V_Y. The system as shown is stable for positive values of V_X if the amplifier gives an overall sign reversal. For negative values

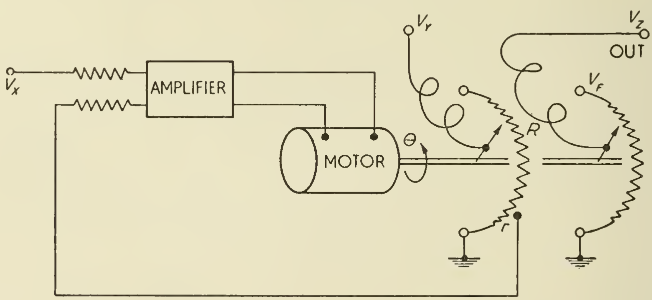


Fig. 76. Alternative Arrangement for Divider

of V_X, the fixed voltage $-V_F$ must be replaced by $+V_F$, or else the number of amplifying stages must be changed to give no phase reversal.

The frequency response of this kind of divider is limited by the available motor acceleration, as in the case of the electro-mechanical multiplier, so that the most favourable application is to the division of two voltages which vary only slowly, with a limited range of values of constant sign for the divisor voltage.

An alternative connection for a divider is shown in fig. 76, the two potentiometers being interchanged compared with fig. 75. The feedback voltage is seen to be $V_Y r/R$, which is equal to $V_Y r\theta_o/\theta R_o$, and when the motor comes to rest this is balanced by the input voltage V_X, so that

$$V_X = -V_Y r\theta_o/\theta R_o.$$

The output voltage V_Z is given by

$$\begin{aligned} V_Z &= \theta V_F/\theta_o \\ &= -V_F V_Y r/V_X R_o \\ &= KV_Y/V_X. \end{aligned}$$

This arrangement has no obvious advantage over that of fig. 75 except that the sign of the output can easily be changed, and it has some disadvantages. As shown, the system is stable only when V_X and V_Y are of opposite sign, and if V_X and V_Y are of the same sign either a sign-reversing amplifier must be used, or a different number of stages must be employed in the amplifier. Instability will occur if the slider moves on to the "r" section of the first potentiometer; for suppose V_Y has a negative fixed value and V_X is positive and increases steadily. To provide a steadily-increasing feedback voltage to balance V_X the value of θ must be steadily reduced, but when the slider reaches the tapping point the feedback voltage is equal to V_Y, and can increase no further. If $|V_X|$ is larger than $|V_Y|$, the motor will continue to turn in the direction of decreasing θ until the limit of travel is reached.

Another type of electromechanical divider can be formed by using the second potentiometer of fig. 75, without the V_Y input, as a variable input resistor for a high-gain amplifier with a fixed feedback resistor. The overall gain of this arrangement is then inversely proportional to V_X.

The disadvantage of some of these dividers, that they can handle divisors of only one sign, is generally unimportant, because it is over-ridden by their inability to operate with divisors near zero. It is unusual to find, in any practical problem, a divisor which can have both positive and negative values, but no values near zero.

If in the arrangement of fig. 74 the terminal of the multiplier marked V_X is connected to the output terminal of the amplifier, so that $V_X = V_Z$, and if the gain of the amplifier is very large,

$$V_Z^2 = V_Y \text{ very nearly,}$$

or,

$$V_Z = \sqrt{V_Y}$$

so that this combination of multiplier and amplifier can be used as a square-root device. Any of the multiplier types already described can be used in this way, and the biased-diode type of multiplier to be described in Section 7.5 can also be used. There is, however, a more direct way of computing square roots with biased diodes which will be mentioned in this later Section.

7.4 CURVE FOLLOWERS AND FUNCTION GENERATORS

A curve follower, or function generator, is a device which, when fed with an input voltage V_1 gives as output a voltage V_O such that $V_O = f(V_1)$, where f is some predetermined function, usually single-valued but not necessarily monotonic. The functions which are most commonly used in this way are sines and cosines, parabolas, and functions expressed as curves derived from experimental results.

Several curve followers have been developed using cathode-ray

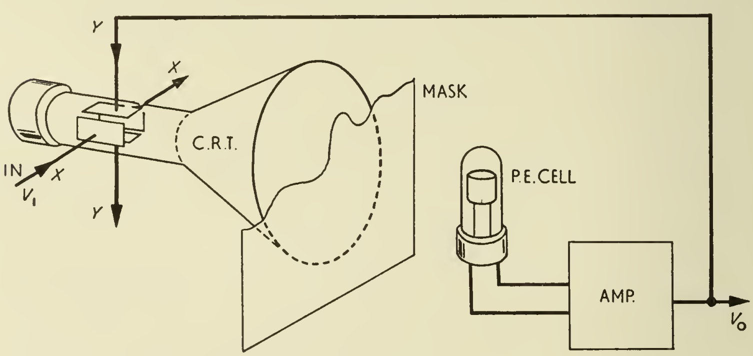


Fig. 77. Function Generator

tubes and photocells, and fig. 77 shows the principle of one of these invented independently and almost simultaneously in 1946 by D. J. Mynall (Ref. 33) and D. M. MacKay (Ref. 34), and later developed by Squires and Pieper.

The function to be reproduced is first drawn as a graph on thin card, to a suitable scale. Then the part of the card above the curve is cut away, leaving a silhouette type of "mask", as shown in fig. 77. Alternatively, the part of the card below the curve is painted black, and a photograph is taken on a glass plate, giving a mask which is clear glass above the curve and opaque below.

The mask is fixed close to the screen of the cathode-ray tube, with the axes of the graph parallel to the X and Y deflection directions of the tube, and a photocell is placed behind the mask as shown. Thus, when the spot of the cathode-ray tube screen is above the edge of the mask its light falls on the photocell, but when the spot is below the edge the light is cut off. The output from the cell is passed through an amplifier, and the amplified voltage is applied to the Y-deflection plates of the tube with such a sign that the spot is deflected downward. The spot cannot be deflected below the edge of the mask, because then there would be no light falling on the cell, and the deflection voltage would disappear, and with a suitable value of amplifier gain the spot comes to rest with half its area showing above the edge of the mask. This condition can be maintained almost independently of the height of the mask, or of any X-deflection which may be present. Suppose now that the graph from which the mask was made is $y=f(x)$, and suppose an input voltage V_1 is applied to the X plates of the tube. This will give a deflection in the X direction proportional to V_1, equal to $k_1 V_1$, say, but because of the action of the mask, cell, and amplifier, there will also be a deflection in the Y direction given by

$$y = f(k_1 V_1).$$

The voltage to give this deflection is the output voltage V_O of the amplifier, and $y=k_2 V_O$, where k_2 is a constant very nearly equal to k_1. Hence,

$$V_O = \frac{1}{k_2} f(k_1 V_1)$$

and with an appropriate choice of scaling factors the output voltage represents the desired function of the input voltage.

For various practical reasons, such as non-linear relation between deflection voltage and spot deflection, lack of uniformity of the screen, non-orthogonality of the two pairs of deflecting plates, etc., it is difficult to reduce the errors with this device to less than about 2%, and furthermore, operation becomes uncertain if the slope of the graph is greater than about 80°. However, the time of response can be made very short, so that the device can be used in repetitive simulators, and the function being generated can easily be changed by inserting a different mask.

Another curve follower using a cathode-ray tube and photocell has been designed by R. H. FORREST and K. H. TREWEEK (Ref. 35). This has the facility for following a line on a transparent plate or film, and does not need the area below the line to be opaque. The arrangement is generally similar to that shown in fig. 77, but a small alternating potential is applied to the Y plates, in addition to the normal deflecting voltage. This alternating potential elongates the spot vertically to a length slightly greater than the thickness of the line on the film. Thus, the output of the photocell is an alternating potential, and if the upper part of the spot shows above the line this potential is in phase with the potential applied to the Y plates, whereas if the lower part of the spot shows below the line the phases are opposite. The photocell output is passed to a phase-sensitive rectifier and then amplified, which gives deflecting voltages which can be used to bring the centre of the spot always very close to the centre of the line.

Besides its use as a function generator this device can also be used as a film "reader". Suppose a record has been made of variation of some quantity with time by photographing the spot of a c.r. oscilloscope on a steadily-moving photographic film, or by some similar method. Then, with no potential on the X plates the film can be moved steadily in the X direction between the c.r. tube and photocell of the Treweek device, and the output voltage of the amplifier will be proportional to the quantity originally recorded. This arrangement could be used, for example, for inserting "road bumps" into the simulator suggested for investigating the suspension of road vehicles (Section 4.3), or for injecting the effects of variable wind into simulators for ballistic and aeronautic problems.

In another device of this kind developed by LANGE and HERRING the function is again recorded as a black line on a transparent film, but there is no feedback from the photocell output to the deflecting plates. Instead, a saw-tooth voltage is applied continuously to the Y plates, so that the photocell gives a pulse every time the spot passes the line on the film. Another pulse is given when the spot passes a reference line on the edge of the film, and the time interval between the two pulses is proportional to the value of the function, provided the saw-tooth is linear. This arrangement has the advantage that there is no limit to the slopes of functions which can be followed, and in particular stepped waves create no difficulty.

A number of different electro-mechanical function-generators have been devised, usually making use of an amplifier, motor, and feedback potentiometer to give a shaft position proportional to an input voltage, as for the multiplier of fig. 71. The linear potentiometer P_2 is replaced by one with a suitably "shaped" winding fed from a constant voltage, so that the voltage on the slider represents the desired function of the shaft position. This arrangement is not easily adaptable for a wide range of functions, because of the difficulty of winding the potentiometers, but it is very suitable for generation of sine and cosine functions, using sine-cosine potentiometers.

Sine-cosine potentiometers are of three main types; the "flat-card" type, the "shaped-card" type, and the "mechanical linkage" type. In the first type a thin resistance wire is wound uniformly on a flat parallel-sided card or strip, and the ends are connected to points at fixed equal positive and negative potentials. The slider is carried on a spindle whose axis is perpendicular to the plane of the card, and in

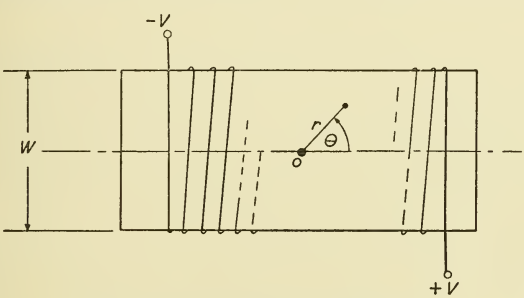


FIG. 78. Sine-Cosine Potentiometer

line with the centre of the winding. The arrangement is illustrated schematically in fig. 78, which shows that the number of turns between the centre O and the slider is proportional to $\cos\theta$. The voltage at the slider, however, is not exactly proportional to $\cos\theta$ because the slider does not remain on the centre line of the card; for example, when $\theta = 90°$ the potential at the end of the slider is not zero, but differs by the voltage drop along the part of a turn between the slider and O. This inherent error can be kept small by using many turns of fine wire, and also by making the width W of the card large compared with the length of the wiper arm. The use of a large number of turns also helps to reduce the size of the voltage jump as the wiper passes from one turn to the next, but there will inevitably be errors due to the finite number of turns, the most obvious of these, though not necessarily the most serious, being in the region of $\theta = 0$ and $\theta = 180°$, where the motion of the slider is almost parallel to the wire, and the slider may stay in contact with one particular turn over appreciable angles. There is a tendency for this turn and a few adjacent turns to be displaced by the slider, and it is important that the winding should be carefully done and the turns cemented firmly to the card.

Apart from these considerations the accuracy of this device depends chiefly on the linearity of the winding, i.e. the constancy of the voltage drop per unit length of wire and of the number of turns of wire per unit length of card. Allowing for all causes the error will not be much below 1% unless great care is taken, but the comparative simplicity of the device makes it attractive for applications where moderate accuracy suffices. In the form shown in fig. 78 the device is a cosine potentiometer, giving a slider voltage proportional to $\cos \theta$, but the same device can be used as a sine potentiometer, giving an output proportional to $\sin \phi$ if $\phi = 90° - \theta$.

Attempts have been made to produce a sine/cosine potentiometer on the principle of fig. 78 but with the wire winding replaced by a uniform layer of carbon-based resistance material, conducting strips the full width of the card being fixed at opposite ends and connected to the fixed supply so as to give a uniform fall of potential from one end to the other. Although this arrangement is cheap it is difficult to make the carbon layer sufficiently uniform and resistant to wear.

Another form of this same basic type of potentiometer is shown in fig. 79. Here the resistive element is provided by a square dish of conducting liquid, such as copper sulphate solution, and conducting plates P_1, P_2 occupying the full width of the dish are connected to voltages $\pm V_F$ to provide a uniform fall of potential. The "slider" is a metal probe P_3 on the end of an arm carried by a vertical shaft. The liquid electrolyte is not very convenient, but the output impedance of the device is low, and there is no "noise", no wear of the resistive element, and no discontinuity in the output voltage due to "wire-to-wire" jumps. A second probe, displaced 90° from the first, can be used to give a quadrature output. A device of this kind, driven at constant speed by a motor, has been successfully used to generate a sinusoidally-varying output voltage for examining the response of electro-mechanical systems to sinusoidal disturbances.

In the "shaped-card" type of sine-cosine potentiometer (fig. 80) the wire is wound on a strip of card which is bent into the shape of a cylindrical surface, and one edge of the card, on which the wiper

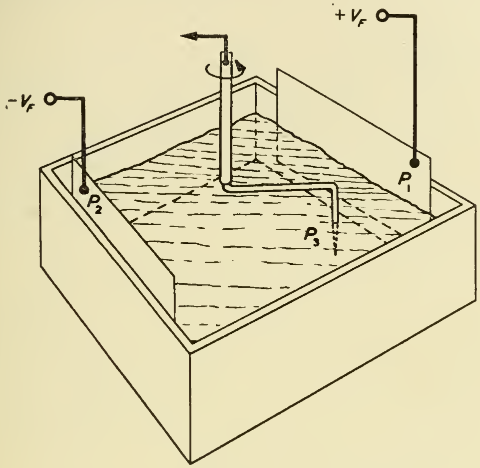


FIG. 79. Liquid Potentiometer

runs, is in a plane perpendicular to the axis of the cylinder, exactly as for an ordinary potentiometer. The other edge of the card, however, is shaped so that when the ends of the winding are connected to a source of constant potential the potential at the slider

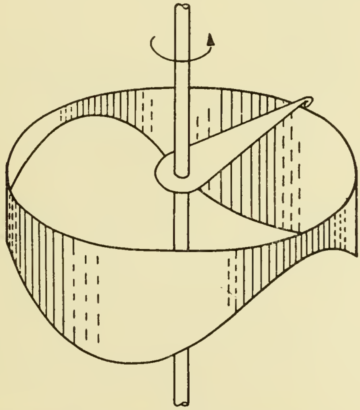


FIG. 80. Shaped-Card Potentiometer

represents the sine or cosine of the shaft angle. Potentiometers of this type have been made in large sizes, capable of great accuracy and a high degree of resolution (Ref. 36), but they have not so far found wide application in general purpose analogue computers.

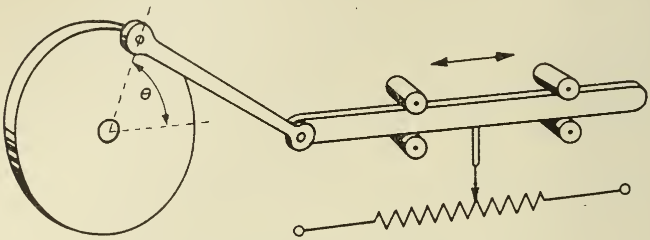


FIG. 81. Crank and Connecting Rod

Of the various mechanical linkages that have been used in conjunction with linear potentiometers the most obvious is the simple crank and connecting rod, shown schematically in fig. 81. This is simple, but does not give a good approximation to sinusoidal motion unless the connecting rod is very long.

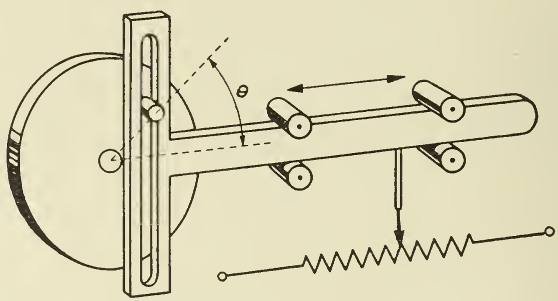


FIG. 82. Scotch Yoke

Another mechanical sine-linkage is the "scotch yoke", shown in fig. 82. The longer part of the T-shaped member runs in guides, while the cross member has a slot in which runs a crank pin, fitted to an arm or disc, whose angular position represents θ. Ideally, this arrangement can give exact sinusoidal motion, but in practice it is difficult to avoid backlash due to the imperfect fit of the pin in the slot.

An interesting linkage due to E. A. JOHNSON, which involves no sliding guides, is illustrated in fig. 83. A shaft S_1 has a hole, perpendicular to its length, in which runs a pin attached to the forked end of a rod R. The lower end of the rod runs in a bearing at the end of an arm carried on a second shaft S_2. The axes of the two shafts are perpendicular, and the extension of S_2 meets S_1 at the pin hole.

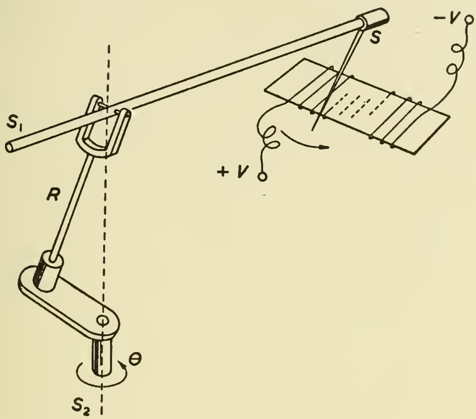


FIG. 83. Sine Linkage without Slides

Thus, if shaft S_2 turns, shaft S_1 rocks, and a sinusoidally-varying voltage is given by a slider S which makes contact with the edge of a flat potentiometer winding, as shown. This arrangement gives an effective length of slider arm which varies with the angle ϕ by which

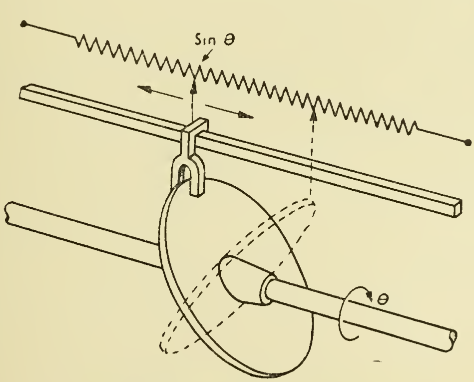


FIG. 84. Swash-Plate Mechanism

S_1 is displaced from its mean position, and this is necessary because it is tan ϕ, and not ϕ itself, which is proportional to sin θ or cos θ.

Another linkage, which is capable of good accuracy, and which is used in the TRIDAC computer (Section 11.3) is the "swash-plate" device shown in fig. 84. A flat circular plate is fixed to a shaft in such a manner that the axis of the shaft passes through the centre of the plate, but the axis is inclined at an angle of, say, 45° to the surface of the plate. Round the edge of the plate runs a collar, retained by grooves or flanges so that the collar is free to turn relative to the plate, in the manner of a strap on an eccentric sheave. A lug is fixed to the collar and constrained by guides to move parallel to the axis of the shaft, and the slider of the potentiometer is attached to the lug. The mechanical errors in this arrangement can be kept to 0·1% if the design and manufacture are of the highest quality, though it is not easy to achieve corresponding linearity in the winding of the potentiometer unless it is fairly large. By modifying the lug and slide arrangement a number of sliders can be driven by a single swash-plate mechanism, so that a number of different quantities represented by V_1, V_2, etc., can be simultaneously multiplied by $\cos\theta$ or $\sin\theta$ if $\pm V_1$, $\pm V_2$, etc., are applied to the separate windings of potentiometers driven by the swash plate. Furthermore, a second lug and potentiometer slider can be attached to the collar, displaced 90° from the first, so that multiplication by both $\cos\theta$ and $\sin\theta$ can be accomplished with one swash plate. In TRIDAC, one swash plate drives 12 sine potentiometers and 12 cosine potentiometers (fig. 143).

Sine and cosine potentiometers of the types described give an output voltage proportional to $V\sin\theta$ or $V\cos\theta$, where V is the voltage applied across the winding and θ is the angular position of a shaft. When driven by a "position servo", as used in fig. 71, they can be used to produce the sine or cosine of an angle represented by the input voltage to the servo. They can also be used with a knob and dial on the input shaft for manual setting of θ, and this is useful when both $\sin\theta$ and $\cos\theta$ are needed (e.g. in fig. 38) since by ganging a sine potentiometer and a cosine potentiometer the two functions can be set by a single control. Interesting and sometimes useful results can be obtained by using a sine or cosine potentiometer as the feedback potentiometer in the arrangement of fig. 71.

Besides the mechanisms described, devices depending on cams have also been used; but for sine and cosine variations they are usually less satisfactory than other methods. For non-sinusoidal variations, however, cams have been used to limited extent, though they are difficult to make accurately, and the need for a fairly high pressure to keep the follower in contact with the cam tends to increase the driving power required.

Another device which is sometimes useful for the approximate generation of functions is shown in fig. 85*a*. This is a potentiometer

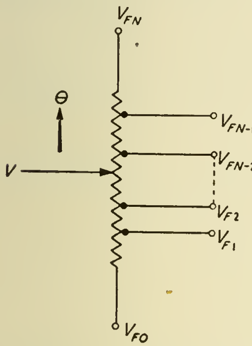


FIG. 85*a*. Approximate Function Generator

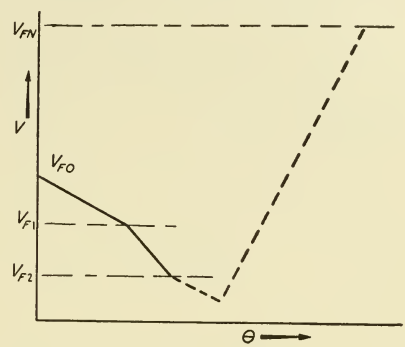


FIG. 85*b*

with a number of taps which are connected to a set of fixed voltages V_{F0}, V_{F1}, V_{F2}, . . . V_{FN}. Between any pair of taps the voltage varies linearly with the rotation θ of the shaft, so that the complete graph of slider voltage against θ is made up of a number of straight lines, as in fig. 85*b*. The intervals in θ are fixed by the positions of the taps, but the slopes of the straight-line segments are fixed by the differences $(V_{F1} - V_{F0})$, etc., and these can be varied at will, subject to the limitation of permissible dissipation in the winding. Some rounding of the sharp discontinuities in the curve can be achieved by fitting two wipers to the potentiometer, displaced from each other by a suitable small angle, and taking the output from the centre tap of a resistor connected between them. Calculation of the performance of this circuit is very tedious unless the source impedances associated with the fixed voltages V_{F0} etc. are negligibly small, which will not usually be the case, and it is simpler to make the final adjustments by trial and error.

7.5 BIASED-DIODE DEVICES

Many applications have been made of thermionic valves in which the shapes of characteristic curves, such as anode current/grid voltage curves have been used to produce desired non-linear effects, but it is difficult by such means to achieve accurate and stable results because of the effects of supply voltage changes and variations between different valves of the same type. The current tendency is to use arrangements in which the valve characteristics have at most only a minor effect, and this applies particularly to a class of non-linear devices which use biased diodes and fixed resistors. A description of this sort of arrangement was given by DEELEY and MACKAY in 1949 (Ref. 37), though the basic idea was known earlier (Ref. 49). The following account is based mainly on the work of BURT and LANGE.

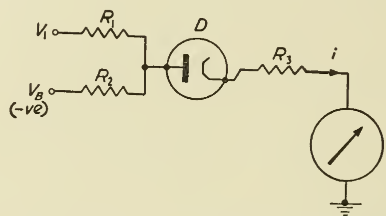


FIG. 86a. Biased-Diode Circuits

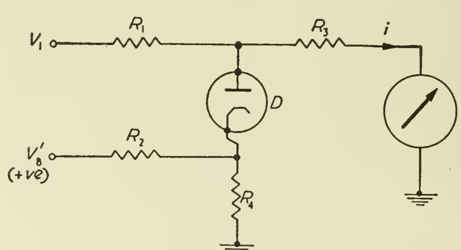


FIG. 86b

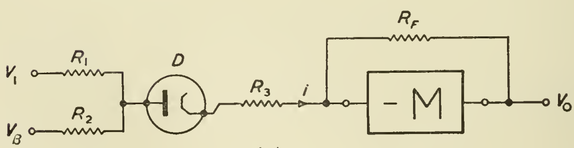


FIG. 86c

The basic principles of the biased-diode device are illustrated in fig. 86, in which V_1 is the input voltage, V_B is a bias voltage, and the output quantity is the current i flowing in R_3. In fig. 86a, suppose that V_B is negative and V_1 increases steadily from zero, and assume that the diode impedance is either zero or infinite, according as the anode is positive or negative with respect to the cathode. At first the anode of the diode will be negative, so no current will flow in R_3, but when V_1 is sufficiently positive the diode will start to conduct, and thereafter the output current will increase in proportion with V_1. It is easily seen that current flows when

$$V_B + \frac{R_2}{R_1 + R_2}(V_1 - V_B) > 0$$

i.e. when

$$R_1 V_B + R_2 V_1 > 0.$$

The value of the current is

$$i = \frac{\dfrac{R_1 V_B + R_2 V_1}{R_1 + R_2}}{\dfrac{R_1 R_2}{R_1 + R_2} + R_3} = \frac{R_1 V_B + R_2 V_1}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

The curve relating i and V_1 is shown at (a) in fig. 87.

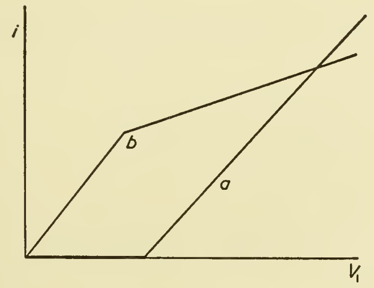


FIG. 87

In fig. 86b V_B' is positive, and again V_1 may be supposed to increase from zero, so that initially the diode does not conduct, and a current i flows, given by

$$i = \frac{V_1}{R_1 + R_3}$$

When V_1 reaches a value such that

$$V_1\left(\frac{R_3}{R_1 + R_3}\right) - V_B\left(\frac{R_4}{R_2 + R_4}\right) > 0$$

the diode begins to conduct, so that a re-distribution of currents occurs, and the current in R_3 rises less rapidly.

When the diode conducts the current is:

$$i = \frac{R_4}{R_3+R_4}\left\{\frac{V_B+\left(\dfrac{R_2}{R_1+R_2}\right)(V_1-V_B)}{\dfrac{R_1R_2}{R_1+R_2}+\dfrac{R_3R_4}{R_3+R_4}}\right\}$$

$$= \frac{R_4(V_BR_1+V_1R_2)}{R_1R_2(R_3+R_4)+R_3R_4(R_1+R_2)}$$

This gives a characteristic of the type shown at (*b*) in fig. 87.

In practice the output is usually required as a voltage rather than as a current, so the meter of fig. 86*a* is replaced by a high-gain amplifier with a feedback resistor, as shown in fig. 86*c*, and correspondingly for fig. 86*b*. It is easily shown that when the gain M is very large the output voltage is

$$V_O = -iR_F.$$

By reversing the signs of V_1 and V_B and reversing the diode connections, in either fig. 86*a* or 86*b*, negative output current can be produced, and by combining several elementary circuits a characteristic can be produced which consists of a number of straight-line

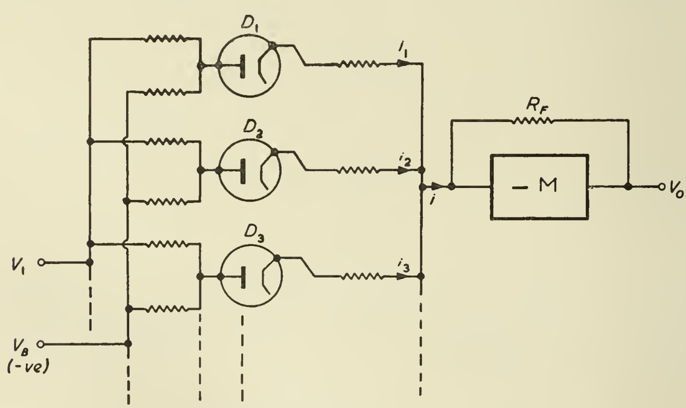


FIG. 88. Multiple Biased-Diode Circuit

segments whose slopes and lengths can be adjusted so that they form a series of chords or tangents or secants to a given curve. One simple combination, shown in fig. 88, includes a number of circuits of the type shown in fig. 86*a*, and the resistance values are chosen to give a different voltage for each diode, so that as V_1 rises from zero the diodes start to conduct successively. Thus the total current rises more and more steeply as more and more diodes conduct, and the output voltage changes in a corresponding way. Curves for the first three diodes are shown in fig. 89.

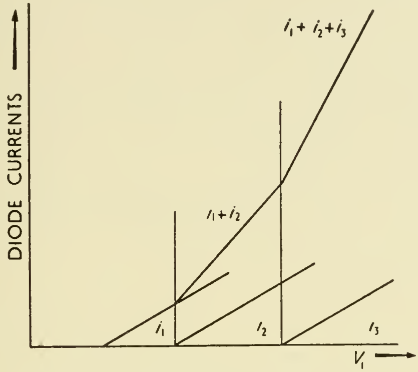


FIG. 89. Diode Currents in Fig. 88

If more diodes are used, and the biasing voltages set closer together the straight segments of the combined characteristic become shorter, giving a closer approximation to a smooth curve. In principle the errors due to finite length of the straight segments can be reduced to any desired extent by using more diodes, but in practice the forward impedance of the diodes, variation of diode impedance with heater voltage, etc., set a limit. However, it is possible without great difficulty to achieve errors of about 1%, and if great care is taken in selecting, ageing, and matching the diodes, and if wire-wound resistors are used, with carefully chosen bias values, errors as small as 0·2% are possible.

Curves representing almost any single-valued function can be produced by this technique, and two of the most important are the parabola and the sine. The curve of fig. 89 indicates how the positive half of a parabola $y = kx^2$, or $V_O = -kV_1^2$, can be produced, with the circuit shown in fig. 88, which is repeated in the upper half of fig. 90. The negative half is produced by an identical circuit, but with a sign-reversing amplifier inserted between the input voltage V_1 and the resistance network, as shown in the lower half of fig. 90. When V_1 is positive V_1' is negative so that the diodes D_N do not conduct, but the diodes D_P behave as those in fig. 88. If the sign of V_1 is changed from positive to negative with no change in magnitude, V_1'

will have the same sign and magnitude as V_1 before the change, so that diodes D_N will conduct, diodes D_P will be cut off, and the output voltage will be unchanged. In practice, of course, it would probably

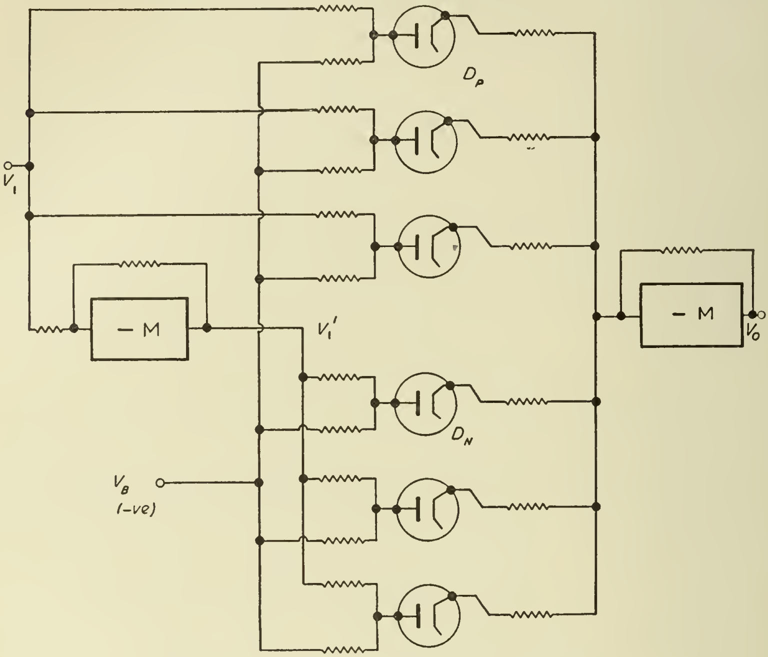


FIG. 90. Generator for Parabolic Function

be necessary to use more than three diodes for each side, depending on the desired accuracy and the range of voltages to be covered.

An important application of the "squaring" circuit of fig. 90 is in the biased-diode multiplier, which makes use of the relation

$$xy = \tfrac{1}{4}\{(x+y)^2 - (x-y)^2\} \tag{41}$$

In this multiplier the squaring circuit is used to produce voltages proportional to $(V_1 + V_2)^2$ and $(V_1 - V_2)^2$, and for this purpose the basic circuit of fig. 86*a* is modified by the addition of another input resistor, R_1', as shown in fig. 91*a*. If $R_1' = R_1$, a simple application of THEVENIN's theorem (Ref. 38) shows that V_1, V_2, R_1, R_1', may be replaced by a voltage $\tfrac{1}{2}(V_1 + V_2)$, applied through a resistor $\tfrac{1}{2}R_1$ (fig. 91*b*). If more diodes and resistors are added, as indicated in fig. 92, the output voltage V_O is proportional to $-(V_1 + V_2)^2$, so

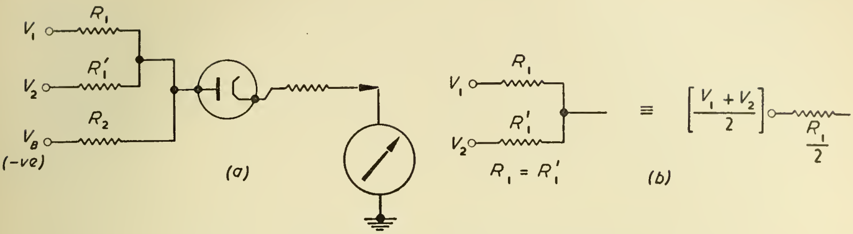


FIG. 91. Biased-Diode with Two Input Voltages

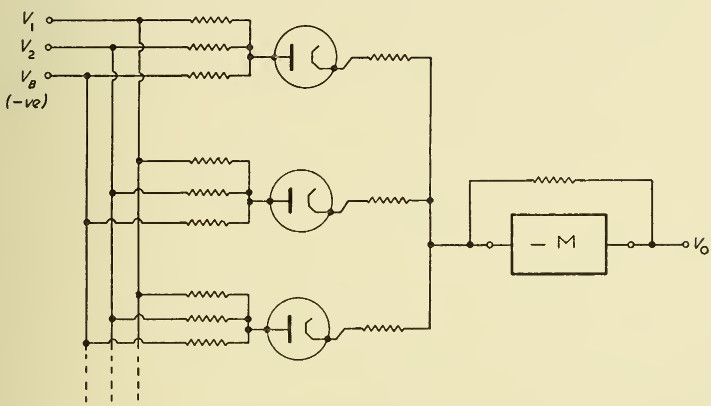


FIG. 92. Squared-Sum Generator

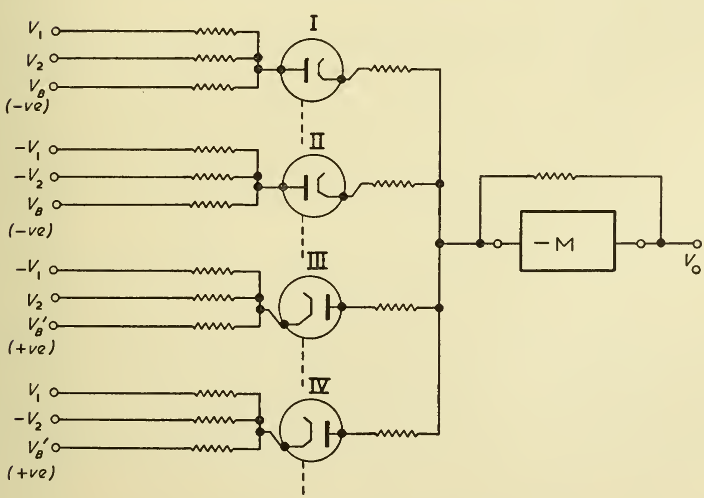


FIG. 93. Biased-Diode Multiplier

long as $(V_1 + V_2)$ is positive. To handle negative values of $(V_1 + V_2)$ an identical set of diodes and resistors is used, but with input voltages $-V_1$ and $-V_2$, produced by means of sign-reversing amplifiers. These two sets of diodes and resistors are indicated respectively by sections I and II of fig. 93, where for clarity only one diode-resistor combination of each set is shown, and the sign-reversing amplifiers for $-V_1$ and $-V_2$ are omitted. If a third identical set of diodes and resistors were added, with input voltages V_1 and $-V_2$, this would add to the output voltage a component proportional to $(V_1 - V_2)^2$, assuming $V_1 - V_2$ to be positive, but equation (41) shows that this component is to be subtracted from the $(V_1 + V_2)^2$ term, not added to it, and this could be achieved by inserting a sign-reversing amplifier between the common cathode resistor connection for this set of diodes and the input of the summing amplifier N. There is, however, a simple method of reversing the sign of the $(V_1 - V_2)$ voltage, which involves only reversing the signs of all the input voltages, including V_B, and reversing the connections of all the diodes. This gives the arrangement indicated by section III of fig. 93. The diodes of this section can conduct only when $(-V_1 + V_2)$ is negative, or when $(V_1 - V_2)$ is positive, and the current flowing in the diode anode resistor is reversed in direction compared with sections I and II. For negative values of $(V_1 - V_2)$ a fourth set of diodes and resistors, with input voltages $+V_1$ and $-V_2$ are needed.

The complete arrangement, as indicated in fig. 93 thus gives an output voltage

$$V_O = -k'\{(V_1 + V_2)^2 - (V_1 - V_2)^2\} = -k_1 V_1 V_2,$$

where $k = 4k'$ is a scaling factor which depends on the choice of the values of the resistors including, of course, the feedback resistor of the feedback amplifier.

An arrangement of biased diodes and resistors may be used to give an output proportional to $\sin\theta$, where $\theta = V_1/K$ and V_1 is an input voltage. Such an arrangement is shown in fig. 94. Since $\sin\theta$ has a finite slope at the origin the first element in fig. 94 is a single resistor, R_1, which would by itself give the straight line OP in fig. 95. A set of diodes and resistors indicated by I in fig. 94, and fed with an input voltage $-V_1$ from a sign-reversing amplifier gives a current i_2 of opposite direction to the current i_1 in R_1, so that the total current rises less rapidly as V_1 increases. As more diodes of the set become conducting i_2 increases more rapidly, and when V_1 exceeds a value $K\pi/2$ i_2 is increasing more rapidly than i_1, so that the total current begins to fall. When $V_1 = K\pi$ the currents i_1 and i_2 are equal, so the sum, and hence V_O, are zero. This arrangement will give $V_O = -k \sin (V_1/K)$ over the range $\theta = 0$ to $\theta = \pi$, and could

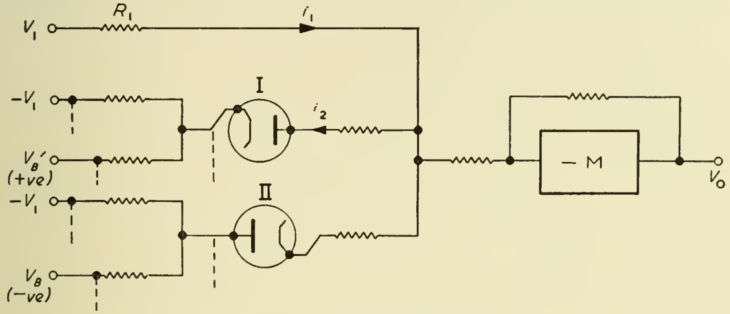


FIG. 94. Sine Generator

be extended, if desired, for larger angles. However, when $V_1/K = \pi$ all the diodes in set I are conducting, and they would continue to conduct for higher values of V_1, so that the total current would be the algebraic sum of a fairly large number of individual currents and it would become increasingly difficult to maintain the required accuracy. It is preferable, if a range of 2π for θ is needed, to use

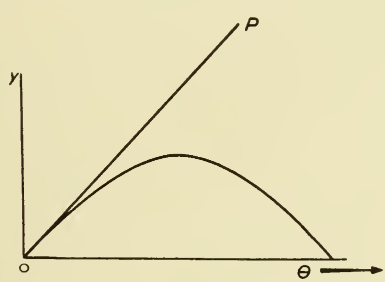


FIG. 95

$-\pi$ to $+\pi$ rather than 0 to 2π, since a second set of diodes, indicated at II in fig. 94, can produce the required output voltage for negative values of V_1, with all the diodes of set I in the non-conducting state. The performance of the complete circuit of fig. 94 is represented by

$$V_O = -k \sin (V_1/K) \quad -\pi \leqslant V_1/K \leqslant \pi,$$

as shown in fig. 96a. If an output of opposite sign is desired, as shown in fig. 96*b*, this requires only the reversal of signs of the V_1 voltages in fig. 94, giving $-V_1$ into R_1 and $+V_1$ into I and II. Since both $+V_1$ and $-V_1$ are already available, no additional equipment is required.

When a biased-diode arrangement is being used to represent some function involving fairly rapid changes of slope it is sometimes

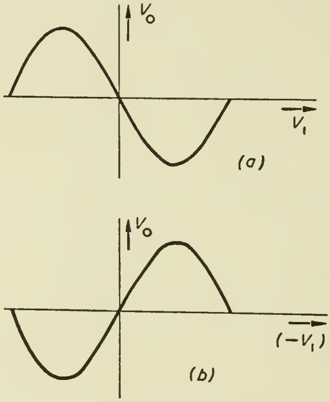


FIG. 96. Negative-Sine and Sine Functions

possible to achieve a useful rounding of the rather sharp discontinuities which appear at the junction of the straight lines by using an artifice suggested by R. N. KIRKNESS. This requires simply the addition of an alternating potential V_{ac} to the normal "d.c." input voltage V_1. The frequency of this potential should be fairly high, and care must be taken that it has no adverse effect in the succeeding sections of the simulator. The peak-to-peak amplitude is best found by experiment, but is generally less than the interval between successive bias voltages. When the value of V_1 is such that the swing of V_{ac} does not extend over more than one straight line segment the mean value of the output voltage is unchanged, but if part of the positive half cycle extends, say, to a segment of higher slope the mean output is somewhat higher than if V_{ac} were absent. As V_1 increases the proportion of the cycle which extends on to the higher slope also increases, until V_1 reaches the bias value, when the increase due to V_{ac} is a maximum. As V_1 increases still further the effect of V_{ac} diminishes until, when the negative peak of V_{ac} does not swing below the bias point the effect of V_{ac} is again zero.

The effect is illustrated in fig. 97, for a function represented in the absence of V_{ac} by the two lines OA, AB, and for a particular value V_1' of the input voltage V_1. When V_{ac} is present the mean output will be increased by the difference between the two shaded areas, and the characteristic will be as $OA'B$.

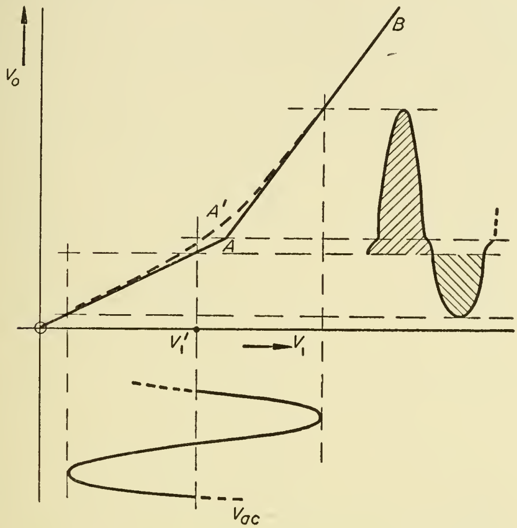


FIG. 97. Super-Imposed Alternating Potential

Besides the parabolas and sine functions described, biased diode arrangements can be used to provide many other curves, using methods similar to those already outlined. The diode circuits are suitable for operation at relatively high frequencies, if required, since there are no moving parts and no reactances except diode and stray capacitances and possibly some small reactance in the resistors and wiring. The advantages of versatility, quick response, and potential accuracy, however, are somewhat offset by the complexity of equipment. Thus, for the highest accuracy a complete multiplier of the type shown in fig. 93 might include 40 or 50 diodes, and 200 resistors; and in order to produce $\pm V_1$, $\pm V_2$, and possibly $\pm V_B$, up to six high-gain d.c. amplifiers may be needed, besides the one shown, the exact number being dependent on whether any of the required voltages are available from low-impedance sources.

Another curve which can be produced without difficulty by means of biased diodes is the upper half of the parabola

$$V_1 = kV_O^2.$$

This gives an output voltage proportional to the square root of the input voltage, so that the arrangement can be used as a square-root device, and it is often preferable to the type described at the end of Section 7.3.

7.6 BACKLASH AND DEAD-SPACE SIMULATION

For a few elementary non-linear functions very simple biased-diode arrangements are possible. The best-known of these is probably the limiting circuit, shown in fig. 98*a*. So long as $-V_B < V_1 < V_B$ neither diode conducts, and $V_O = V_1$. When $V_1 > V_B$ the lower diode conducts, and assuming its impedance and also the impedance of the source of V_B are very low compared with the resistor, the anode voltage will not rise appreciably above V_B, so that $V_O = V_B$. Similarly, when $V_1 < -V_B$, $V_O = -V_B$. The relation between V_O and V_1 is shown in fig. 98*b*. Asymmetrical limiting can easily be achieved by using unequal bias voltages, and the sharpness of the "knees" in the characteristic can be

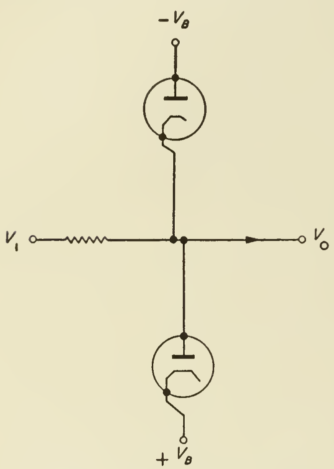


Fig. 98*a*. Biased-Diode Limiter

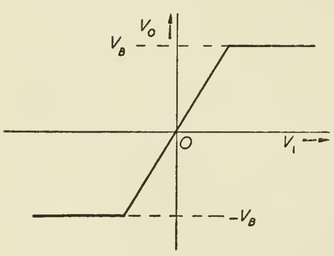


Fig. 98*b*

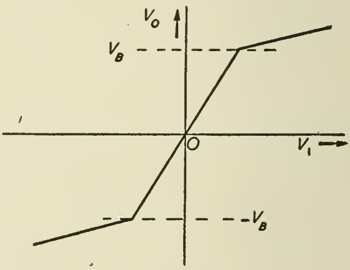


Fig. 98*c*

controlled to some extent by altering the value of the resistor. A characteristic of the shape shown in fig. 98*c* in which the slopes of the outer parts are not zero, but have particular values, can be achieved by including resistors in series with the diodes.

Another simple arrangement of biased diodes is the "dead-space" circuit shown in fig. 99*a*. If $-V_B < V_1 < V_B$ neither diode conducts, so $V_O = 0$. If $V_1 > V_B$ the upper diode conducts, and if its impedance and the impedance of the bias battery are negligibly small, $V_O = V_1 - V_B$. Similarly, if $V_1 < -V_B$, $V_O = V_1 + V_B$, and the complete characteristic is shown in fig. 99*b*. The sharpness of the knees will

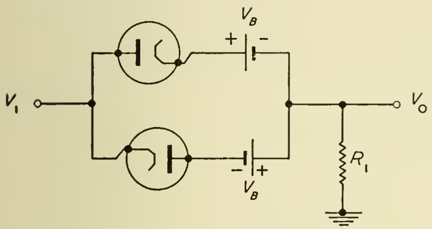


FIG. 99*a*. "Dead-Space" Generator

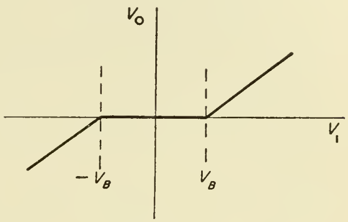


FIG. 99*b*

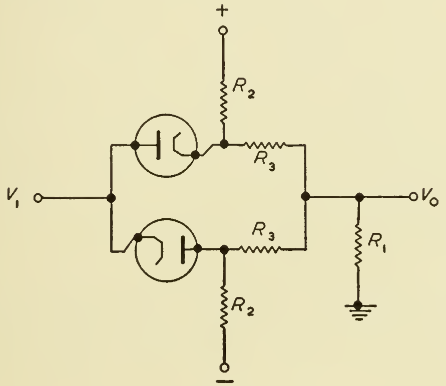


FIG. 99*c*

depend on the relative impedances of diode, battery and resistor R_1. The use of batteries may be inconvenient, because the voltage can only be adjusted in steps, and because the value of V_B will change if current flows for appreciable periods; if so the bias voltages can be supplied from other sources via resistors R_2, R_3, as shown in fig. 99*c*. This circuit, however, introduces some attenuation, so that when $V_1 > V_B$, for example, $V_O = k(V_1 - V_B)$, where $k < 1$. The circuits of fig. 99*a* and 99*c* can be modified to give a characteristic in which the slope of the centre section is positive instead of zero by adding resistors in parallel with the diodes.

Besides limiting and dead-space effects it is sometimes required to simulate backlash. This is not easy to do in the general case, but the arrangement of fig. 99 can be used in some cases. Suppose, for example, that the system being simulated includes the simple mechanism shown in fig. 100. A pin P at the end of an arm pivoted at P_1 is a loose fit in a fork F at the end of a second arm pivoted at

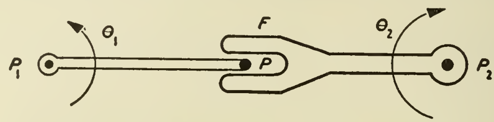


FIG. 100. Mechanisms with Backlash

P_2. The pin and fork may be taken as representing gear-wheel teeth. In the position shown, take $\theta_1 = \theta_2 = 0$, and now assume that there is a spring-centring device which always tries to return θ_2 to zero. If θ_1 begins to increase from zero θ_2 does not change until the pin strikes the upper prong of the fork, and thereafter, assuming the effective lengths of the arms remain equal, $\theta_2 = \theta_1 - \alpha$, where α depends on the diameter of the pin and the spacing of the prongs. When θ_1 begins to decrease this relation is maintained until $\theta_1 = \alpha$, when $\theta_2 = 0$, and if θ_1 continues to decrease through zero, θ_2 remains zero until $\theta_1 = -\alpha$ and for values of θ_1 less than $-\alpha$, $\theta_2 = \theta_1 + \alpha$. The relation between θ_1 and θ_2 is therefore of the form shown in fig. 99*b*, so that the mechanism could be simulated by the circuits of fig. 99*a* or 99*c*.

A more usual form of backlash occurs when a frictional force operates at the pivot P_2 so that the fork only moves when it is pushed

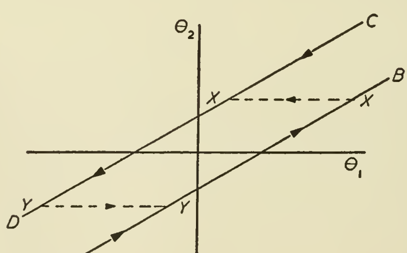


FIG. 101. Backlash Characteristic

by the pin. If θ_1 at first increases θ_2 will not change until the pin hits the upper prong of the fork, and then it will increase steadily with

θ_1; and if θ_1 later reverses its direction the fork will remain stationary while the pin moves from the upper prong of the fork to the lower. This type of motion is represented in fig. 101, where AB gives the relation between θ_1 and θ_2 while θ_1 is increasing, and CD gives the relation while θ_1 is decreasing. This representation applies only while the pin remains continuously in contact with one prong of the fork, and during the change-over from one prong to the other, when θ_2 remains constant, the relation between θ_1 and θ_2 is represented by a horizontal line, such as XX or YY, between AB and CD. Any electronic device to represent this behaviour requires some sort of "memory" which "remembers" the value of θ_2 during the change-over period, and one such arrangement, which is satisfactory in some

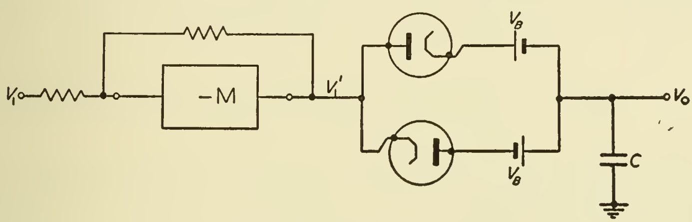


FIG. 102. Backlash Circuit

cases, is shown in fig. 102. The pair of biased diodes operates in the same way as those in fig. 99a, but the resistor is replaced by a capacitor, and the input voltage is applied through a high-gain amplifier with feedback. Suppose initially that the voltages V_1 and V_O are zero, and then V_1 begins to decrease steadily. The voltage V_1' increases steadily but V_O remains at zero until V_1' overcomes the bias on the upper diode, and then the diode conducts, so that the capacitor is connected via the bias battery to the amplifier output terminal. The object of the amplifier is to provide a source of very low impedance to feed the capacitor so that no appreciable lag is introduced. In practice the resistance of the battery is liable to be greater than the amplifier impedance, but if it is, say, 10 ohms, and the capacitance is $1{\cdot}0\mu F$ there will be a lag of 10 micro-seconds, which will not usually be important. Thus, ignoring the small voltage drop in the conducting diode, $V_O = V_1' - V_B$, and this relation will be maintained so long as V_1' continues to increase. If, however, V_1' begins to decrease, the upper diode will cut off, since there is no means of removing charge from the capacitor, and the output voltage will therefore remain at the value it reached when V_1'

reversed. However, when V_1' has fallen by $2V_B$ the lower diode will begin to conduct, and thereafter the output voltage will decrease with V_1' in accordance with $V_O = V_1' + V_B$. The modification which avoids the use of bias batteries is shown in fig. 103, but it may be necessary, in using this circuit to take account of the attenuation introduced by the resistor network.

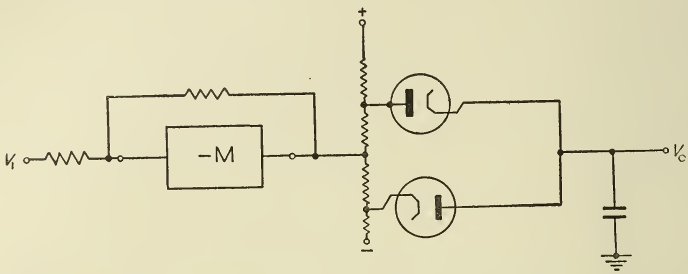


FIG. 103. Backlash Circuit without Separate Batteries

With ideal diodes, a capacitor of infinite leakage resistance, and infinite input impedance for the device into which V_O is fed, this arrangement would give the characteristic of fig. 101; but in practice the diodes do not cut off sharply, they have appreciable forward impedance, and there will be some leakage from the capacitor during the change-over intervals. These effects may be serious if it is required to represent a sharp change from, say, AB to XX, or if the system is expected to spend appreciable periods in the condition where the pin (fig. 100) is not touching either side of the fork. In other cases, however, this arrangement gives a satisfactory simulation of this type of backlash.

7.7 USE OF CONTINUOUS ALTERNATING VOLTAGES

A number of function generators have been made which use continuous sinusoidal voltages. Two of these, a sine-cosine generator and an arc-tangent computer, developed by SUTCLIFFE and others, will be briefly mentioned.

In the sine-cosine generator a saw-tooth wave and trigger circuit are used to give a square wave whose mark/space ratio varies with an input voltage V_1, in the same manner as for the variable-mark-space multiplier (Section 7.1). This square wave is used to "gate" a sinusoidal voltage whose frequency is simply related (say equal or double) to the frequency of the saw-tooth, the gating being so arranged that the wave is transmitted during the "mark" and suppressed during "space". The resultant slices of sine wave are smoothed, and it can be shown that the output voltage, after subtraction of a constant, is proportional to sin $(V_1/k+\phi)$, where k is a constant and ϕ is a phase angle which can be changed by changing the relative phase of saw-tooth and sinusoidal waves. When $\phi=0$ a sine function is produced, and when $\phi=90°$ the function is a cosine.

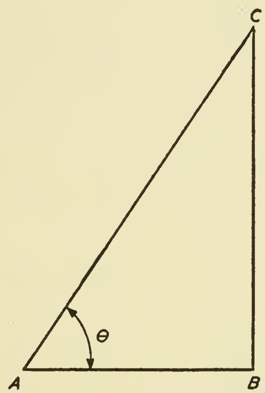


FIG. 104

The arc-tangent computer gives an output voltage V_O such that $V_O=K\tan^{-1}(V_1/V_2)$, where K is a constant, V_1 and V_2 are two input voltages representing two known sides of a right-angled triangle (BC, AB, fig. 104), and V_O/K is the angle θ which it is required to compute. The two voltages are used to modulate two carrier waves, of the same frequency but in phase quadrature. The two modulated waves are combined, giving a wave of amplitude proportional to $(V_1^2+V_2^2)^{\frac{1}{2}}$, and of phase $\tan^{1}(V_1/V_2)$ relative to the V_2 carrier. This wave is amplified, clipped by biased-diode limiters, and differentiated, giving a series of pulses whose position in time, relative to a second series of pulses derived by amplifying, clipping, and differentiating a reference wave, is proportional to $\tan^{-1}(V_1/V_2)$. The pairs of pulses are used to operate a "flip-flop" circuit, giving a square wave whose mark/space ratio is dependent on $\tan^{-1}(V_1/V_2)$, and a voltage proportional to this angle can be obtained by clamping and filtering this wave.

7.8 THE TRIGGER CIRCUIT

A number of different trigger circuits have been designed (Ref. 28) and fig. 105 shows a popular arrangement due to Schmitt (Ref. 39).

In the normal condition, with the input terminal at about earth potential, V_2 passes a moderate current, and this gives sufficient potential drop in the variable resistor to bias V_1 almost to cut-off. If the input terminal is made more positive the current through V_1 increases and the grid of V_2 becomes more negative, so that the cathode of V_1 becomes less positive and there is a further increase of current through V_1. If the input terminal becomes steadily more positive a critical value is reached at which a small change in the grid potential of the V_2 tends to produce a somewhat larger change, of the same sign, in the anode potential of V_1. The circuit is then unstable, and the anode current of V_1 increases very rapidly until limited by non-linear effects. At the same time the current through V_2 falls almost to zero.

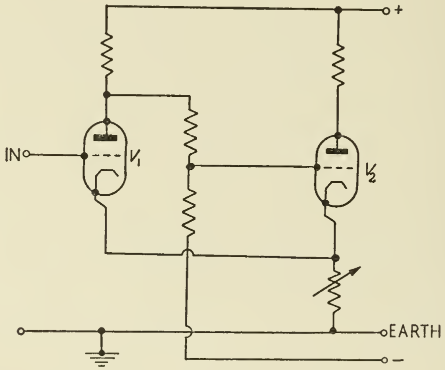


Fig. 105. Schmitt Trigger Circuit

If the input terminal is now made steadily more negative another critical point will be found at which the circuit reverts suddenly to its original condition, with moderate current in V_2 and very little in V_1. The difference in voltage between the two triggering points may be 20 volts or more, or as little as 0·1 volt, depending on the circuit values, especially the value of the variable resistor.

8

USING SIMULATORS

When a given dynamic system is to be studied by means of a simulator it is convenient to derive a set of equations which describe as far as possible the behaviour of the system, although complete description by means of equations may not be possible because of non-linearities. In the initial stages some of the non-linearities and possibly all of them may be ignored, and some of the less important dynamic effects, such as small time lags, resilience of shafts, etc., can also be omitted. If it is intended that the simulator shall ultimately include actual parts of the real dynamic system (Section 10.3) these should, in the initial stages, be simulated by simple lags or in some other convenient manner. As the work proceeds, more complete and accurate representation may be required, and appropriate changes are made to the equations, and additions to the simulator, so that at least some of the non-linearities and smaller time lags, etc., are included. It is important to remember, however, that however far this procedure is taken, a completely accurate simulation of a dynamic system is not possible. For example, the resilience of a shaft may have been ignored; but if it has been taken into account, it may appear only as an elastic effect, with no account taken of the inertia of the shaft itself, which may or may not be negligible compared with the inertia of an attached gear wheel; and a more realistic representation still would allow for the fact that the mass of the shaft is not "lumped" but "distributed", so that the shaft acts as a mechanical "transmission line". Furthermore, the elastic behaviour of the shaft may be non-linear, and so on, and corresponding imperfections appear in all other practical components.

For some cases, of course, such refinement of representation as these remarks suggest may be quite unnecessary and even undesirable because of the wasteful use of computing elements and the extra computing errors introduced by these elements. For other cases it may be necessary to include a large number of these minor effects. In this respect the simulator method is no different from other computing techniques, but experience has shown that it is rather easy, when using a simulator, to fall into the way of thinking that a particular simulator, especially if it contain many elements, is a true model of the dynamic system being studied. In fact, the simulator is a representation of some other dynamic system, which resembles the real dynamic system more or less closely, but which contains a number of idealizations, omissions, and approximations. The art of using a simulator consists, at least in part, in being able to judge the degree of perfection in simulation which a given dynamic system and a given problem require; in continually remembering that the simulator model is imperfect; and in bearing in mind the imperfections when the results of simulator work are being studied.

Assuming, however, that a decision has been made as to which effects are to be included and which are to be ignored, attention can be given to "setting up" the simulator.

8.1 SETTING UP A SIMULATOR

In setting up a simulator or analogue computer a dominant requirement is to choose the scale factors in such a way that no overloading occurs, but at the same time the voltages representing variables should always be as large as convenient in order to prevent undue contamination by noise, hum, etc. If the simulator is to be used to represent a given set of linear equations, or a set of equations with a number of non-linearities of types for which suitable curve followers or function generators are available, the construction of a block diagram is usually not difficult. In some cases there will be more than one possible block diagram, and the choice will depend on the number and types of computing elements available, on the range on variables to be handled, and the accuracy required. To take a simple example, suppose it is desired to compute the side BC in the right-angled triangle of fig. 104, when voltages representing AB and θ are available. If it is known that θ will always be small, it may be sufficient to use the approximation $\theta = BC/AB$, using a multiplier to give $BC = \theta . AB$. Otherwise, BC could be computed from $BC = AB . \tan\theta$, by using two or three terms of the tangent expansion;

$$\tan\theta = \theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \ldots$$

using summing amplifiers and multipliers to generate θ^3 and θ^5, or using biased-diode function generators to produce cubic or fifth-power curves in the same manner as for a parabola. For larger values of θ an arc-tan computer of the type described in Section 7.7 could be used, or a tangent curve could be produced by one of the several available curve followers or function generators.

When the block diagram has been drawn a list is prepared of the expected or estimated maximum values of quantities which are to be represented as output voltages from computing elements. Generally, of course, the maximum values of all the quantities will not be known, but for those which are known a first estimate of scale factor is made by dividing the output voltage which the element can give without overloading by the maximum value of the corresponding variable. Thus, suppose the expected maximum value of an angle θ is 12°, and the maximum output voltage is 45 volts. The scale factor will be $K_\theta = 45/12$ volts per degree $= 3{\cdot}75$ volts per degree. Suppose also that the maximum value of another angle ϕ is 14°, with the same maximum of 45 volts, so that the first estimate of scale factor is $K_\phi = 45/14 = 3{\cdot}22$ volts per degree. In practice it would usually be preferable not to use these two scale factors, but to use a common factor of 3·0 volts per degree for both variables, because the round number simplifies mental estimates, and the use of a common factor eases comparisons and reduces the chance of errors due to using the wrong scale factor. Using 3·0 volts per degree the θ voltage will only reach 36 volts, compared with the permissible 45, but the gain in convenience will offset the small loss of signal-to-noise ratio. The scale factors thus derived will probably indicate the gains required in some of the amplifiers, so that values of input and feedback resistors can be found. For amplifiers whose gains cannot be found in this way because maximum values of some variables are not known, an estimate is made, based on the broad assumption that the output voltage of one amplifier is the input to another similar amplifier, so that it is reasonable, in the absence of any other indication to choose a gain of unity for an amplifier with one input, and a rather smaller value if there are two or more inputs. For the time constants of an integrator, there may be some information about expected values and manner of variation of input quantities, but if not an estimate may be made, based on whatever is known of the time scale of the dynamic system being represented.

On the basis of estimates of this kind, and corresponding estimates for multipliers, function generators, etc., the scale factors for the remaining quantities in the problem can be found. The estimates, of course, will often be very rough, but they will enable a complete set of scale factors, gains, time constants, etc., to be written down and entered on the block diagram. It is important at this stage to check for consistency; for example, if a summing amplifier is being used to compute $\theta = \phi + \psi$, then with the chosen input and feedback resistors an input voltage representing, say, one degree of ϕ according to the scale factor chosen for ϕ should give an output voltage representing one degree of θ according to the scale factor for θ, which may or may not be the same as for ϕ. Similarly for ψ, which may have a different scale factor again, and hence may need a different value of input resistor. For an integrator computing θ from $d\theta/dt$, an input voltage representing a step of, say, one degree per second according to the scale factor for $d\theta/dt$ must produce an output voltage which rises steadily at a rate of K volts per second, where K is the scale factor associated with θ.

When this first block diagram is complete, with resistor and capacitor values, etc., inserted in accordance with the estimates, the simulator elements are adjusted and interconnected to correspond, and the desired disturbing voltages, such as steps or sine waves are applied. The output voltages from the computing elements are observed by means of voltmeters, recorders, or oscilloscopes, and if any of the elements overload, or if the maximum output voltage from an element is much smaller than the overload value, appropriate changes are made to gains, time constants, etc., and the consequent changes of scale factor are noted. When the changes have been made the simulator will be capable of accepting the required inputs without error due to overload or to excessive noise.

This procedure for setting up the simulator and deciding the scale factors is not difficult to apply to a small machine, but may become tedious when applied to a large machine. However, a large simulator is very often set up in a number of stages, perhaps by beginning with a highly simplified model of the complete dynamic system and making a series of modifications which give a more accurate and complete representation of the system; or alternatively, it may be possible to consider the dynamic system in a number of sections, for which separate small simulators are set up and tested before being connected together to form the complete simulator. Such methods not only simplify the problem of deciding scale factors, but they help the operator to build up a mental picture of the mechanism he is studying; in fact a stage-by-stage procedure is often justified for this reason alone, even though, as is usually the case, the operator of a large machine has had experience in setting up and using smaller machines.

The most common type of input is the "step" voltage, and in systems which contain non-linearities it must be remembered that a change in the amplitude of the input step will not cause a proportional change in the amplitudes of all the other voltages in the simulator. In other cases the amplitude of the step may not be critical, and slight overloading may be removed by using a smaller step. Other input functions are the impulse function, sinusoidal voltages, and random functions such as noise. The impulse function, although important as a mathematical concept, is not often usable in analogue computers because of the probability of overloading. The ideal impulse has an amplitude approaching infinity and lasts for an infinitesimal time, the product of amplitude and duration being finite. The practical impulse has finite amplitude and duration, but to give a reasonably close representation of an ideal impulse the duration must be short compared with the time constant of the system to which it is applied. To satisfy this requirement without using an amplitude so large as to cause overloading usually means that the disturbing effect on the system is small, and the resulting output voltages are also small and difficult to measure accurately. For this reason and also because a practical approximation to an impulse is more difficult to produce than a step function, the impulse function has not found wide application in analogue computers.

Testing with sinusoidal voltages is a useful and generally straightforward technique, though care must be taken to avoid overloading when the frequency is changed, because though the amplitude of the input may be the same the effects of resonance may give unexpectedly large voltages in some parts of the system. Disproportionately large errors can be caused if a sharp peak of voltage draws grid current in an input circuit and allows charge to appear on a capacitor which has only a high-resistance discharge path when the grid current stops. The charge on the capacitor may persist for long enough to give appreciable error, even though the period of true overload is negligibly brief.

The most useful application of sinusoidal voltages is to complex linear systems. The sinusoidal response of simple linear systems can be calculated with relatively little labour by the method developed for communication networks and servomechanisms (Refs. 40, 41); and the response of non-linear networks to sinusoidal inputs is liable to be misleading unless great care is taken in its interpretation. If frequencies of less than a few cycles per second are required some difficulties arise in the generation of suitable voltages, and although a number of successful methods have been devised, equipment for the generation of sine waves of, say, ten cycles per second down to a few cycles per minute is not so readily available as for audio and higher frequencies (see Section 9.2).

With random inputs occasional overloads for very short periods cannot always be avoided. Indeed, if the input voltage has a "normal" distribution there is a finite, though usually small, probability that any finite amplitude, however large, will occur; but in practice the input voltage will have been produced by some equipment which is itself limited in the amplitudes and rates of change of amplitude, etc., which it can handle, so that it is usually necessary to accept an amplitude distribution which falls somewhat short of the ideal. Even so, if the simulator includes any differentiating sections, either as explicit differentiators or as loops in which, for example, an integrator appears as a feedback element, the differentiated random voltage may include large peaks, and there may be danger of overloading.

8.2 INITIAL CONDITIONS

Before a computation on a simulator or differential analyzer can be begun all the voltages representing variables must be set to the correct values, in accordance with the set of initial conditions associated with the problem. This applies even for voltages whose initial values are zero, since there may be stray voltages due to residual charges on capacitors, etc. For this purpose, and also to facilitate "zero-setting" in amplifiers not fitted with automatic means for drift correction, it is common practice to provide each computing element with a "start" relay whose prime function is to disconnect the input terminal of the element temporarily from its source of input voltage and connect it to earth. The arrangement for an amplifier is shown in fig. 106, in which the input resistor is earthed so that when zero-setting has been done the output voltage is zero. This condition is maintained for all the amplifiers in the simulator up to the instant when the solution is begun, when the closing of a master switch operates all the relays simultaneously and all the input resistors are connected to their appropriate sources of input voltage. Following the usual practice the relay contacts in the diagrams are all shown in the "unoperated" condition.

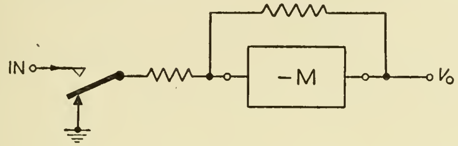


FIG. 106. Amplifier with Start-Relay Contacts

Fig. 106 gives the condition $V_O = 0$ when $t = 0$. For any other initial value the arrangement of fig. 107 is used, the second input resistor R_B being earthed until $t = 0$ when the changeover contact applies a steady voltage V_B, giving an initial value for V_O equal to

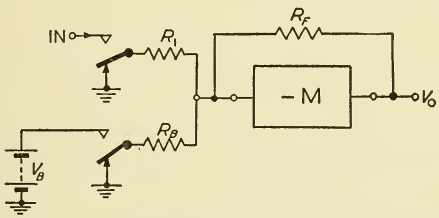


FIG. 107. Start-Relay for Non-Zero Initial Value

$-V_B R_F / R_B$. A similar method can be used with other computing elements whose output/input relations are independent of time.

For an integrator a relay is also used, and operated by the same master switch, but for the condition $V_O = 0$ at $t = 0$ there must be no

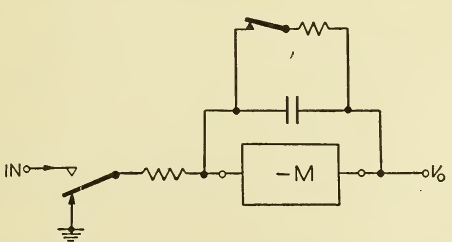


FIG. 108. Integrator with Start-Relay Contacts

charge on the capacitor at $t = 0$, so the relay is arranged to shunt a low-value resistor across the capacitor as well as earthing the input resistor, until $t = 0$, as shown in fig. 108. The low-value resistor is included to limit the capacitor current and to reduce sparking.

If the conditions require that the integrator output voltage V_O has some value other than zero at $t=0$, i.e. if there is a non-zero constant of integration, the initial value can be introduced by several different methods, most of them depending on producing an appropriate charge on the capacitor at $t=0$. An obvious way of doing this is shown in fig. 109, where relay contacts are used to connect a battery directly to the capacitor. At $t=0$ the contacts are opened and the capacitor voltage is then V_B. It is easily shown that this gives V_O an initial value equal to $V_B M/(1+M)$, which is effectively equal to V_B if M is large. A limiting resistor of low value is connected in series with the battery.

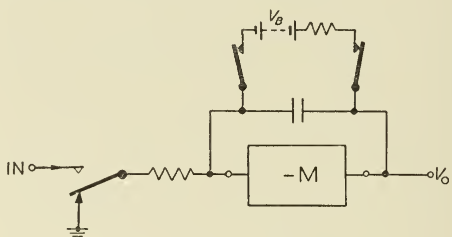


FIG. 109. Initial-Condition Switching for Integrators

The arrangement of fig. 109 has the disadvantage that a separate isolated battery must be provided for each integrator. Methods can be devised, as in fig. 110, which allow the capacitors of several

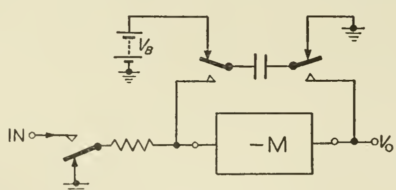


FIG. 110. Initial-Condition Switching for Integrators

integrators to be charged from a common supply, but in the arrangement shown the amplifier has no feedback impedance for $t<0$, and the very high gain may give rise to undesirable spurious output voltages. Extra relay contacts may be provided to connect a temporary feedback resistor, but careful timing of the switching is necessary to ensure that the capacitor is connected into circuit shortly before the feedback resistor is disconnected, so that there is no interval with no feedback impedance, and also to avoid any appreciable discharge of the capacitor via the temporary resistor during the short period when both are connected.

Another method of putting an initial charge on the capacitor is shown in fig. 111. Here the total integrator capacitance is formed of two separate capacitors which are connected in parallel when integration begins. Before $t=0$, however, one of the capacitors is connected to a source of appropriate voltage and when the relay operates the capacitors are connected in parallel and the charge is

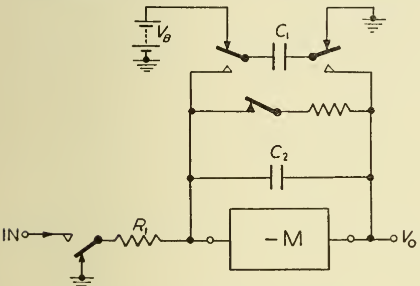


Fig. 111. Initial-Condition Switching for Integrators

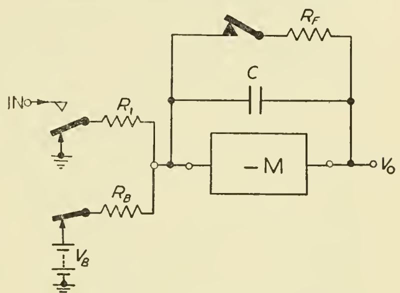


Fig. 112. Initial-Condition Switching for Integrators

shared. If M is very large the initial value of V_O is equal to $-V_B C_1/(C_1+C_2)$. The usual relay contacts are provided to keep C_2 discharged before $t=0$, and it is obviously necessary that these contacts should open before the changeover contacts connect C_1 to C_2.

A scheme with simpler switching is shown in fig. 112. In the condition shown the output voltage is equal to

$$V_O = -V_B \frac{R_F}{R_B}\left(\frac{1}{1+pCR_F}\right)$$

assuming M is very large. Thus the steady-state value of V_O is $-V_B R_F/R_B$, and the capacitor will retain the charge to give this initial value of V_O when the relay is operated at $t=0$. When a computation is complete the relay will be returned to the unoperated condition, as shown in fig. 112, and it is essential that time be allowed for the capacitor voltage, and hence V_O, to approach sufficiently close to their steady-state values before the relays are operated again. This waiting period may be objectionable, though the values of R_B and R_F can be made fairly low so that the time constant CR_F which governs the approach to the steady value may be only a small fraction of the time constant of the integrator CR_1.

A modification of the scheme of fig. 112 is shown in fig. 113, where a second capacitor is shown in parallel with R_B. The relation between V_O and V_B is now

$$\frac{V_O}{V_B} = -\frac{R_F}{R_B}\left(\frac{1+pT_B}{1+pT_F}\right) \tag{42}$$

where $T_B = C_B R_B$ and $T_F = C_F R_F$.

Thus if $T_B = T_F$ the output voltage is always proportional to V_B, and there is no time lag between V_B and V_O. In practice the time

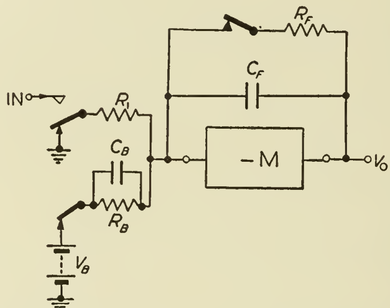


FIG. 113. Initial-Condition Switching for Integrators

constants will probably not be exactly equal, and to examine the effect of inequality suppose first that T_B is the greater, so that $T_B = T_F + T'$, where T' is positive. Then from equation (42):

$$\frac{V_O}{V_B} = -\frac{R_F}{R_B}\left(1 + \frac{pT'}{1+pT_F}\right) \tag{43}$$

When V_B is applied suddenly by the release of the relay after a computation, assuming there is then no charge on the capacitors, the value of V_O will rise suddenly to a value equal to

$$-V_B\frac{R_F}{R_B}\left(1 + \frac{T'}{T_F}\right),$$

which is found by setting $p \to \infty$ in equation (43). This is greater than the steady value in the ratio $(1 + T'/T_F)$, and V_O will settle exponentially to the steady value with time constant T_F. If, on the other hand, T_F is greater than T_B, let $T_F = T_B + T''$ where T'' is positive. Then

$$\frac{V_O}{V_B} = -\frac{R_F}{R_B}\left\{\frac{1+pT_B}{1+p(T_B+T'')}\right\}$$

The quantity in curly brackets can be re-written

$$\frac{T_B}{T_B+T''}\left\{\frac{\dfrac{T_B+T''}{T_B}+p(T_B+T'')}{1+p(T_B+T'')}\right\}$$

so that

$$V_O = -V_B\frac{R_F}{R_B}\cdot\frac{T_B}{T_F}\left\{1+\frac{T''}{T_B}\left(\frac{1}{1+pT_F}\right)\right\} \tag{44}$$

Thus when the relays release immediately after a computation, assuming that there is then no charge on the capacitors, V_O will rise suddenly to a value smaller than the steady-state value in the ratio T_B/T_F, and will approach the final value exponentially with time constant T_F.

When T_B and T_F are unequal and T_B is the greater there may be some slight danger of overloading the amplifier if the inequality is large and if V_O is also large. If the inequality is small, whatever the sign, V_O will instantaneously take up a value very near to the final value, and the exponential change is in the nature of a correction. The time for this correction will be much less than the charging time needed for the arrangement of fig. 112, and a rough measure of the improvement is given by the ratio T_F/T' or T_F/T''. The values of R_B and R_F can again be relatively low, so that T_B and T_F are much smaller than the integrator time constant.

If T'' (or T') is zero, i.e. if $T_F=T_B$, equation (44) represents the ideal form of the preceding method. If T'' is not zero, then the preceding method aims at hastening the approach of the steady state by making T_B, T_F and T'' all as small as possible. An alternative method is to make use of the initial value of V_O, as given by setting $p\rightarrow\infty$ in equation (44), instead of the steady-state value. This requires that the value of V_O as given by equation (44) does not change appreciably during intervals of time comparable with the time occupied by one computation. This may be achieved by making T''/T_B small, so that the relative change from the initial state to the steady state is small, and by making T_F large so that the change is slow. It is therefore desirable to remove the two resistors R_B and R_F of fig. 113, so that T_B and T_F may be as large as possible. In addition, instead of connecting V_B at the end of a computation and allowing the capacitor voltages to "settle" in the interval before the start of the next computation it is now desirable that the capacitors should remain uncharged during this interval,

V_B being connected at $t=0$ or shortly before. This leads to the arrangement shown in fig. 114, where the two capacitors are shunted

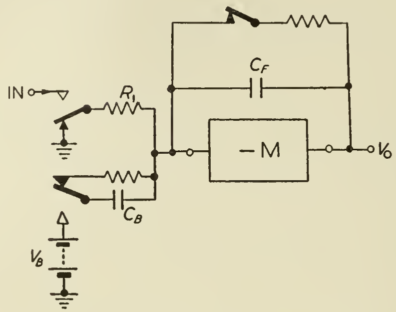


FIG. 114. Initial-Condition Switching for Integrators

through low-value resistors until $t=0$. If the capacitors were ideal components, entirely without resistance, then the relation between V_O and V_B would be

$$V_O = -V_B\frac{C_B}{C_F}$$

which is equivalent to equation (44) with $T''=0$. In any practical use of this scheme, however, the capacitors will not be perfect and an estimate must be made to ensure that the last term of this equation will remain negligible throughout the computation interval.

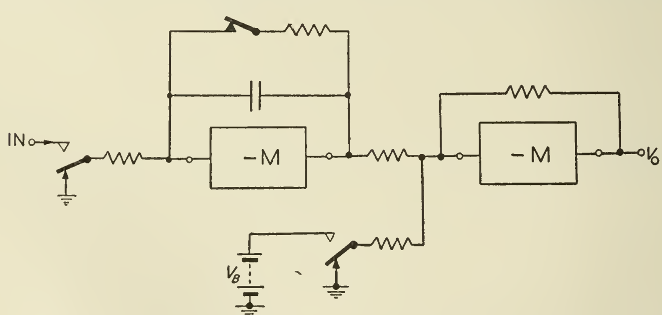


FIG. 115. Initial-Condition Switching for Integrators

A method of adding an integration constant which does not depend on charging a capacitor is shown in fig. 115. Here the integrator is arranged as though the initial value of the output voltage were zero, and the integration constant is added subsequently by a summing amplifier. With this method the full output voltage swing is still available in both positive and negative directions. With other methods, in which the initial-condition voltage appears at the integrator output terminals, the available voltage swing due to the integrating action is reduced in one direction by the amount of the initial voltage.

In all the cases where relay contacts are connected across an integrator capacitor it is important to check that the insulation resistance between the contacts does not appreciably reduce the effective insulation resistance of the capacitor.

For any given simulator the choice of a method of setting initial conditions in the integrators naturally depends on the type and size of the simulator. The most convenient method is undoubtedly that shown in fig. 115. This has no appreciable settling time, gives easy adjustment of the value of the initial voltage, and needs only simple relay contacts with no need for critical timing of the opening and closing of the different contact sets. These advantages are so important that this method has been adopted in a number of medium and large simulators in spite of the cost of extra amplifiers. It is not always necessary to provide each integrator with an additional amplifier exclusively for the addition of the integration constant. It often happens that a summing or reversing amplifier follows an integrator, for some purpose unconnected with initial conditions, and in such cases it can usually be used also for the introduction of the initial condition.

Of the other methods given above some are of little more than academic interest, but one or two are of practical value in some machines when it is desired to avoid the expense of providing additional amplifiers. For a small machine where high precision is not needed fig. 109 may be acceptable, since a few small dry batteries can be used. Fig. 111 needs no appreciable settling time and is convenient to use if each integrator is provided with a suitable relay and a pair of capacitors of equal value. Fig. 114 is specially useful in simulators of the repetitive type (Section 10.2) where the length of the computation interval is known and is usually short.

8.3 SIMULATOR STUDIES

In a complete investigation of a given system the work may be divided broadly into three main parts, although not all of these may be included in any particular study. The first step is to decide whether the system is stable, and in some cases, either because the dynamic system exists and is known to be stable, or because the system is simple enough and sufficiently well understood for a "paper" study to give reliable estimates, no simulator work may be needed to establish stability. In other cases, perhaps because the determination involves the solution of high-order differential equations, a simulator may be used. More commonly, a simulator may be used to determine what sets of values, if any, of the parameters of the system give stability.

For some dynamic systems the simulator work might stop at the first stage, either because there is no further requirement beyond ensuring stability, or because no appreciable adjustment is possible in the value of important parameters of the system. Such a situation might arise if automatic operation or control were being applied to some mechanism or plant which had been designed and built originally for manual operation. Usually, however, when stability has been established, there will be some interest in accuracy, and the second stage of a complete investigation consists in using the simulator to forecast how accurately the dynamic system will perform its task. The errors shown by the simulator will include not only the errors in the system being simulated, but also the errors in the simulator itself. These simulator errors will consist partly of "computing" errors, such as amplifier drift, non-linearity in nominally linear elements, time lag in mechanical elements, etc., and partly of "simulation" errors, due to inaccurate representation of the real system by omission of small resiliences, neglect of small time lags, etc. It is obviously desirable to distinguish the true system errors from computing and simulation errors, but this is not always easy, and sometimes it can only be done by an elaborate series of tests.

When checks of system accuracy are being made by means of a simulator a number of measurements will often be made, using different values of whatever parameters are available for adjustment. This procedure is the beginning of the third stage in the complete investigation, which may be called "optimization". It is obvious that some sets of values of the parameters will give better performance than others, and that one particular set of values will give the best performance. The definition of "best" in this connection is important, and it may refer to the smallest steady-state error; or to the smallest r.m.s. error, especially if there is appreciable "noise" in the system; or to the smallest "dynamic lag" where the input is steadily changing with, say, constant acceleration. Whatever the definition adopted, there will be a "best" set of values for the parameters and the process of optimization includes testing the simulated system with a number of sets of values and deciding which gives the most satisfactory performance. In systems where random signals contribute appreciably to the output voltage the optimization process may take a long time, because the output voltage will be unsteady, and it will be necessary to make observations over a period of time, or to repeat the measurement perhaps dozens of times, before stable mean or r.m.s. errors can be determined for one set of parameters. The whole procedure must be repeated for each set of parameters.

If a sufficiently wide range of values of parameters is tested the "best" set will inevitably be found, but optimization can sometimes be taken a stage further. To illustrate this, suppose the input voltage to a linear system consists of a continuously-varying legitimate voltage plus some noise, both being "stationary" in the statistical sense (Ref. 41), so that they may be considered to have fixed spectral distributions. Then according to WIENER's noise theory (Refs. 42, 43) there is a form for the transfer function of the whole system which will give the smallest r.m.s. error in the output voltage, and if the system is required to give the minimum r.m.s. error its transfer function must conform to the Wiener optimum form. It may be, however, that of all the different transfer functions which can be produced by selecting different sets of parameters, none resembles very closely the optimum shape. The remedy is to add additional elements, such as electrical networks, to modify the response suitably.

Regarding the use of a simulator for investigating the errors in a dynamic system, it is obviously desirable to be able to estimate the degree of accuracy with which the simulator is representing the real system. Reference has already been made to the two classes of simulator error, *viz.* "imperfect simulation" errors and "computing" errors, and to the need for continuous vigilance to ensure that changes in the simulator do not introduce unsuspected new errors. Positive checks on the imperfect simulation errors are generally difficult to devise, and much depends on the skill and experience of the operator. In certain cases, however, checks can be made. Thus if the approximation $\theta = \tan \theta$ is being used to avoid the need for a tangent computer, and if θ is not always so small that the error is clearly negligible, some indication of the magnitude of the error can sometimes be gained by changing either the value of θ or its scale factor by an amount corresponding to the difference between θ and $\tan \theta$ and observing the effect on the output voltage.

Quantitative estimation of the "computing" errors in a simulator is also difficult, and the only thoroughly reliable method is to compare the results given by the simulator with results obtained by accurate step-by-step or analytical solution of the system equations. To some extent, of course, this procedure begs the question, since a simulator is not justified if a paper solution is possible without excessive labour. In practice, however, if large numbers of solutions are required from the simulator it is often quite reasonable to have a few typical cases solved accurately on paper or by means of a digital machine. Another procedure can also be applied when a large number of simulator tests are to be made on a given non-linear dynamic system, with different sets of parameters and perhaps some minor modifications to the system itself. One typical arrangement of the system is selected, with an average set of parameters, and corresponding to this a completely linear system is derived by removing the non-linearities. The equations of motion of this linearized system are written down and a solution is found for the selected set of parameters. The simulator is then set up to represent the linear system, with the selected parameters, and its response is checked against the numerical solution. Any difference will be due to simulator errors, and if these are acceptably small the simulator can then be modified to include the non-linearities with considerable confidence that the results from the non-linear simulator will not be appreciably affected by computing errors. Similar considerations apply to changes of parameters and to modifications of the simulated system, provided these do not depart too much from the original arrangement.

It is, of course, important that the accuracy of individual computing elements should be checked, even when an overall check against a numerical solution is possible. When no numerical solution is available the only method of assessing the accuracy of the simulator results may be to estimate the effect of the errors due to all the elements in the simulator, and often an experienced operator can make a satisfactory estimate. In any case quantitative determination of the error in an element is a necessary procedure, and some reference will be made to it in the next chapter.

In considering the question of the accuracy of the results which a simulator might give it must be remembered that a set of quantitative answers represents only a part of the useful output of a simulator. Another important part of the output is the understanding or "feeling" for the problem which an operator is able to acquire. The simulator is a more or less complete model of the dynamic system he is studying, with the masses, resiliences, and dampings, etc., under convenient control, and in adjusting these quantities and observing the effects on the behaviour of the system he is able to build up, almost subconsciously, an appreciation of the relations between the various quantities and of their relative importance, in a more complete and direct manner than he could from studying graphs and tables of numbers.

Thus, to extract the maximum benefit from a simulator study the operator should be the man who has been engaged in the preceding theoretical studies.

This ideal can only be achieved completely if the problem is of "one-man" size. However, for bigger problems and bigger simulators the same broad principle may be applied, which means that some members at least, of the team concerned, should take part in both the theoretical work and the simulator study. In contrast, an important part of the potential value of the study is lost if the so-called "post-box" method is adopted. According to this method, when the theoretical worker has reached a stage where an extension of his studies by paper methods only would be impracticably laborious and he decides that a simulator should be used, he writes down a formal mathematical statement of the problem, with ranges of values of parameters to be tested, and passes these to the simulator operators. The simulator team perform the necessary solutions, using their simulator only as a calculating machine and not at all as a model of a dynamic system; indeed they may have little or no knowledge of the dynamic system involved. The simulator results, in the form of graphs and tables, are then "posted" back to the theoretical group. Although this method will give the solutions which the theoretical group ask for, it excludes the possibility of direct absorption of understanding of the problem which could be achieved if some members of the team took part in both the theoretical work and the simulator exercise. The method does little towards the solution of one of the great problems consequent on the use of a large computing machine—the problem of digestion by human mentalities of the great mass of information which such a machine can provide as the solutions to complex problems.

The post-box method also implies that the theoretical group can decide in advance the necessary "runs" which the simulator should perform. There is a high probability that any such pre-planned programme will give either too few solutions for sets of parameter values in the critical ranges, or too many solutions with uninteresting combinations of values.

It should be emphasized that the use of simulators is no substitute for thinking. On the contrary more thinking may be needed if a simulator is used properly, because a simulator will only be used for problems which are more complex than can be conveniently handled on paper. The simulator can only perform the unintelligent mechanical parts of the solution of problems. It certainly does not automatically solve the problem confronting the operator. Within its limitations of accuracy, etc., it can solve a problem which resembles the real problem more or less closely, but the degree of the resemblance, and the extent to which the simulator answers may be taken as answers to the real problem, are never complete, though often adequate; and the operator must take care to ensure that they remain adequate. In principle, of course, similar considerations apply to "all-paper" solutions, but experience shows that when using a simulator it is rather easy for vigilance to be relaxed.

When a complex problem is to be solved, even if it is quite certain that a simulator will be used, it is nearly always wise for the operator to spend a while on a paper study. Such a study may need such sweeping simplifications that any numerical answers may be almost useless as even rough solutions. Nevertheless, such a study is of value in giving the operator a chance to think about the whole problem, to make some sort of estimate of ranges of values and relative importance of various quantities, and to decide the stages by which the final complex simulator set-up is to be achieved. At this stage also the operator should think carefully and critically about the aim of the study; he should be quite clear as to what he is trying to discover about his system and in what form the answers are likely to emerge. It is easy to plan to select a large number of values for each of the parameters and to run through all the possible combinations, and to make records of all the variables, but except for quite small problems this will result in a vast indigestible mass of results on paper. A better plan is usually to try a small number of runs, with widely separated values, and to examine the results before performing more runs to fill in the interesting areas. It is often justifiable to record only a few of the variables, for although this entails some risk that information may be lost which might have proved useful, the reduction in the volume of records to be handled is very welcome. The problem of transferring the output of a large computer from paper or film to the minds of the human operators is one which has so far received too little attention. It is false to assume that the amount of information absorbed is proportional to the volume of records.

9

AUXILIARY APPARATUS

The operation of an analogue computer requires, in addition to the computer itself, apparatus for generating the input voltages, observing and recording the output voltages, and for adjusting and checking the performance of the computing elements.

9.1 STEP AND IMPULSE FUNCTIONS

The generation of a step of voltage is very simple, requiring basically no more apparatus than a switch with a clean make or break action. The impulse function, which is occasionally used, can be approximated by a pulse of large amplitude and short duration generated by a pair of electro-mechanical relays, or an electronic "flip-flop" or trigger device, operated preferably by impulses from a standard-frequency source so that the length of the impulse is accurately controlled.

9.2 SINE-WAVE GENERATORS

Sinusoidal voltages can be generated by steady rotation of a sine potentiometer of any of the types, including the liquid type, described earlier (Section 7.4). Function generators of the biased-diode or photo-electric types can also be used over a range $0 \leqslant V_1 \leqslant 2\pi/k$ if they are fed with a saw-tooth input voltage. Thus, suppose such a generator gives an output equal to $V_O = K \sin kV_1$. Then if V_1 rises steadily from zero to a value $2\pi/k$, returns suddenly to zero and then begins to rise again, repeating the cycle indefinitely, then V_O will vary sinusoidally. Alternatively, V_1 can increase steadily from 0 to $2\pi/k$, and then decrease at the same rate to zero. If a biased-diode arrangement is used some economy can be achieved by providing only the zero to π range, and reversing the sign of the output voltage for alternate half cycles. Using a symmetrical input wave, with equal rates of rise and fall it would even be possible to use only a zero to $\pi/2$ range if sign reversing were arranged to give the negative half-cycles.

Some successful sine generators have been made using continuously rotated synchros*. If the rotor of a synchro is fed with current at 50 c/s, say, and turned at a constant rate of 1·0 revolution per second, the current induced in a stator coil will be effectively a 50 c/s carrier with a sinusoidal modulation at 1·0 c/s, and a 1·0 c/s sine wave can be produced by a simple demodulator. Other speeds of rotation give other modulation frequencies, but above a few cycles per second adequate filtering of the 50 c/s carrier becomes difficult, and it may be desirable to use 400 c/s or a still higher frequency for the carrier. An advantage of the synchro method is that a synchro with a pair of stator coils can be used giving two output waves, both of frequency equal to the rotor speed, but in phase quadrature at all frequencies.

For frequencies of a few cycles per second and below electronic generators have not been very widely used. The chief reason for this is the difficulty of limiting the amplitude of oscillation without distorting the wave-form. In the audio and higher frequency ranges two common methods used for generating sine waves are the "LC" oscillator, using conventional tuned networks, and the "RC" oscillator, using a feedback arrangement through either a ladder network or a form of Wien bridge network. In the LC type the filtering action of the tuned circuit is great enough to give a good output waveform even though the anode current of the oscillating valve is badly distorted. In the RC type the usual arrangement is to fit a metal filament lamp or some other variable-resistance device in the feedback circuit so that as the amplitude of oscillation increases the resistance of the lamp also increases, reducing the degree of feedback so that further increase of amplitude is prevented and the amplitude remains steady at some value well below the overload level. Neither of these methods is suitable for much lower frequencies. Tuned circuits are very bulky and of poor performance; and a suitable lamp for an RC circuit must have a thermal time constant equal to at least a period or two of the generated wave, which means that for a generator extending to below 0·1 c/s some minutes might elapse before steady oscillation was achieved. Such a delay is generally unacceptable.

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* "Synchro" is a universal term applied to any of the various synchronous devices used for data transmission, etc. Trade names for such devices include Magslip, Selsyn, Autosyn.

A method of generating slow sine waves which is sometimes useful is to use a pair of high-gain Miller integrators, connected as in fig. 10, but with the $\dot{y}$ feedback omitted, so as to give very light damping. If a step or impulse voltage is applied at x a decaying oscillatory voltage is generated, and with good components the rate of decay can be made slow enough to give a good approximation to a sine wave over useful periods. LANGE, LONERGAN and HERRING, have used this basic arrangement in a successful generator of continuous waves. The peak amplitude is compared with a reference voltage, and once per cycle a pulse of height proportional to the difference is injected into the oscillatory circuit. This not only gives a wave of substantially constant amplitude, but when the generator is switched on the pulse is large, so that the amplitude builds up rapidly.

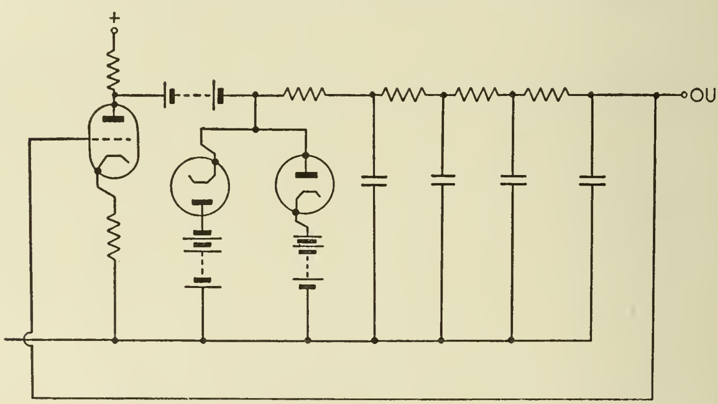

FIG. 116. R.C. Oscillator with Amplitude Limiter

An arrangement proposed by the author for limiting the amplitude in an RC oscillator without undue distortion of the output is shown in fig. 116. This is basically an ordinary ladder-type RC oscillator but with two unusual features. The amplitude of the wave from the anode is limited by a pair of biased diodes; and the network has series resistors and shunt capacitors, the reverse of the usual arrangement. The diodes limit the amplitude but distort the waveform, and the RC network is used not only in its usual role of phase shifter but also as a low-pass filter. This circuit has given oscillations of good waveform at 30 cycles per hour.

Electronic oscillators for "servo" frequencies usually give outputs which have some d.c. content, because of drift in direct-coupled stages. For some purposes this is unimportant, because the method of using such oscillators is often to compare the peak-to-peak voltages at the input and output terminals of the device under test. For other purposes, however, drift correction may be needed.

A non-electronic, arrangement which is sometimes useful is to use an oscillating electromechanical relay to generate square waves and smooth these with a multi-stage RC filter. Harmonic content of a few per cent can be achieved in this way.

9.3 RANDOM VOLTAGES

For the production of random voltages three classes of methods are available, *viz.* amplification and filtering of electrical noise generated in a resistor, valve, or gas-filled tube; rapid switching of a bank of preset potentiometers; and recording, on magnetic tape for example.

The first of these methods involves no new principle. To ensure a random variation without flicker it is usually necessary to start in the audio band or higher, and use a frequency changer to produce noise in the "servo" band. Since the servo band is narrow, in terms of cycles-per-second of bandwidth, high amplification is needed. The waves produced by this method cannot, of course, be repeated, so that to observe the effect of any change in the simulator a large number of runs before and after the change may be needed to give averaged results for which the scatter is small enough for the effect of the change to be detected.

In the switched-potentiometer method the basic arrangement consists of a rotary switch with a large number of contacts, each connected to the slider of a potentiometer fed with a steady voltage. As the switch rotates the voltage on its moving contact jumps from one value to another, depending on the setting of the potentiometers, so that a given wave can be approximated by a stepped wave, the closeness of the approximation depending on the switching rate. Since the output wave is repeatable it cannot truly be called random, but it may be regarded as a section of a longer random wave, and the repeatability is useful, especially in the early stages of an investigation. Different waves can be produced by different sets of settings for the potentiometers.

In place of the rotary switch, the arrangement of fig. 117 has been used. Here the potentiometers are in a square or rectangular array, with terminals and sliders connected as shown. Each slider bus-bar (B_1, B_2 . . .) is connected to the output terminal via a pair of relay contacts, and the relays are operated one at a time, in sequence, by an electronic counting circuit. Only one row of potentiometers is connected to the battery at any one time, the connections being

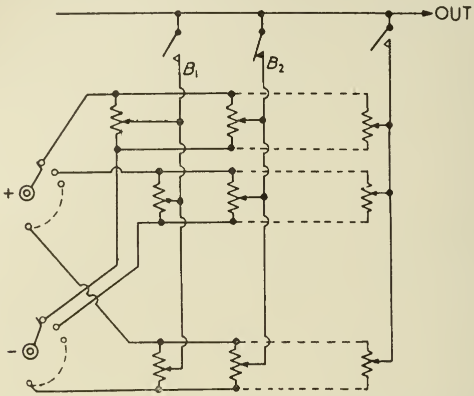

FIG. 117. Sequential Switching of Potentiometers

switched by a uniselector from one row to the next at the end of every cycle of the counting circuit. Thus the potentiometers are, in effect, "scanned" sequentially, giving an output of the same form as the rotary switch.

The voltage waves to be used with these switched devices can be derived from experimental records of noise or other disturbances, or alternatively they may be made up from random number series, with a choice and weighting to give a desired spectrum and amplitude distribution.

Recording devices using magnetic tape, photographic film, or punched paper tape or celluloid film are now well known. With some of these devices, especially those using magnetic tape, the recording of "servo" frequencies cannot be done directly, so that some form of modulation must be used, and this increases the difficulty of achieving accurate reproduction of recorded amplitudes. The input waves to the recorders may be obtained from actual equipment in operation or from a noise generator of any of the types described above.

9.4 PRESENTATION OF OUTPUTS

For the observation and recording of the simulator output voltages standard equipment can generally be used. Meters may be used if

Fig. 118. Plotting Table

the rates of change are slow, and oscilloscopes of the cathode-ray or reflecting-galvanometer (Duddell) type for faster variations. Permanent records may be made by photographing oscilloscope traces or by means of pen recorders, either the usual moving-coil type or the servo-assisted type, which in one form (EVERSHED and VIGNOLLES) has a frequency response extending to well over 10 c/s for amplitudes of three inches peak-to-peak.

For the simultaneous recording of two related variables, such as the co-ordinates of an aeroplane in "eastings" and "northings", a plotting table is commonly used. This is a device in which a pen is carried over a sheet of paper by a carriage which runs on a gantry which in turn runs on a pair of rails fixed to opposite sides of the table. Motion of the pen in two directions at right angles—along the gantry and along the rails—is provided by motors driven from amplifiers. Feedback potentiometers are fitted to the gantry and to the rails so that the co-ordinates of the pen position are accurately proportional to the input voltages to the amplifiers. A plotting table designed at R.A.E. and engineered and manufactured by Dobbie McInnes Limited, Glasgow, is shown in fig. 118. This has a table $18'' \times 30''$, and for slowly-changing inputs the positioning errors are less than 0·25%. The pen will follow a sinusoidal input with a peak-to-peak amplitude of 18″ and a frequency of 0·2 cycle/second with an error of less than 0·5%. At higher frequencies with the same large amplitude the response falls off, partly because of saturation effects, and the pen will follow at somewhat higher frequencies if the amplitude is reduced.

9.5 TESTING AND ADJUSTMENT OF ELEMENTS

For the testing of individual computing elements the methods and equipment depend on the desired accuracy. Summing amplifiers and integrators are usually the most important items. For summing it may be sufficient, if the gain is high enough, to measure the values of the input and feedback resistors, but this method may be inconvenient because it requires disconnection of the resistors. An obvious method is to make use of ordinary voltmeters to measure the output voltage produced by a known input voltage. For the highest accuracy, however, voltmeter methods are not good enough, even though sub-standard meters may be used, and methods based on high-grade resistance boxes are desirable. A simple and accurate method of measuring and adjusting amplifier gain is illustrated in fig. 119. An input voltage V_1 is applied, and a pair of adjustable resistance boxes R_A, R_B, are connected in series between the input and output, the junction of the two resistances being connected to

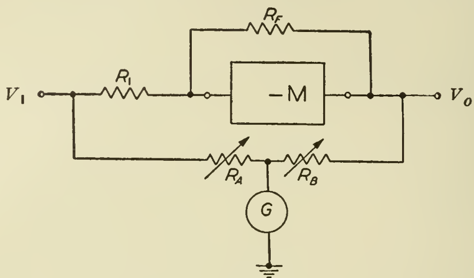

Fig. 119. Measurement of Amplifier Gain

earth via a galvanometer. R_B is set to some suitable value, which is not critical, but should not be so low as to load the amplifier too heavily, and not so high as to give undue leakage errors. R_A is then adjusted for zero current through the galvanometer, and it is easily shown that $V_O/V_1 = -R_B/R_A$. For adjusting V_O/V_1 to a desired value the ratio R_B/R_A is made equal to this value, and then either R_1 or R_F is altered until the galvanometer current is zero. The alteration of R_1 or R_F may not be easy, and it is useful to remember that an ordinary carbon resistor may be added to increase the total resistance by a few tenths of one per cent without seriously degrading the stability of the whole resistance. Thus, suppose a 1,000 ohm carbon resistor is added to a half-megohm wire resistor, giving an increase of 0·2%. If the carbon resistor value changes by as much as 10% the combination changes by 100 ohms, or 0·02%. In principle the same effect could be obtained by using parallel carbon resistors, but the values required would be inconveniently large.

A more convenient method of adjusting the gain of an amplifier is shown in fig. 120. The high-stability resistors R_F and R_1 have values such that the ratio R_F/R_1 is slightly greater than the desired value of overall gain, and the gain is reduced to the correct value by adjusting the potentiometer P. The amount of adjustment will generally not be more than a few per cent, so the fineness of control can be improved by adding a fixed resistor R_P in series with the potentiometer as shown. This arrangement allows rapid setting of the gain if the method of fig. 119 is used to measure the gain. Care must be taken to ensure that the reduced input impedance of the circuit of fig. 120 does not put too great a load on the preceding element.

The method of fig. 119 can be used with different values of "d.c." input voltage, both positive and negative, so that the departure from linearity, measured by the variations of V_O/V_1, can be checked.

For checking integrators the usual methods are based on the simple principle of applying a known input voltage for a known time and

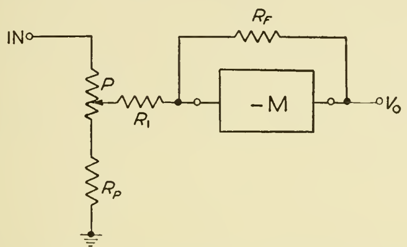

Fig. 120. Adjustment of Amplifier Gain

measuring the output voltage at the end of the measured interval For the most accurate results the input voltage is either derived from standard cells, or is measured by means of an accurate potentiometer using a standard cell as a reference. A very useful arrangement for such purposes is a high-stability "summing" amplifier with a single input voltage from a battery of standard cells. This gives a source of accurately-known e.m.f. which can be made stable to within a millivolt or less, and which is not damaged when appreciable current is drawn. The measurement of the integration interval is best done automatically by impulses from a standard clock, or from a tuning fork or quartz crystal followed by frequency dividers. Precautions must be taken to ensure that the integrating capacitor is free from charge at the beginning of the test. At the end of the integration interval, when the input voltage is removed, the capacitor will retain its charge for a time, and the voltage to which it has been charged may be measured by connecting a high resistance voltmeter across the output terminals. The current drawn by the meter will, of course, hasten the discharge of the capacitor, but because of the very low output impedance of the high-gain amplifier the rate of decay of the voltage will usually be slow enough to allow a measurement to be made. If a meter is not sufficiently accurate, a null method may be used, with the circuit arrangement shown in fig. 121. The potentiometer P is fed from a stable source, and is adjusted so that when the switch S_1 is closed after the end of the integration interval no galvanometer kick is caused. Several trials will be needed before the correct setting of P is found, and the

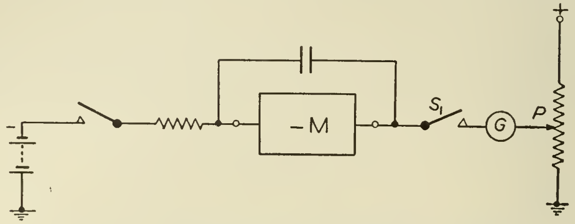

FIG. 121. Measurement of Integrator Voltage

galvanometer should be shunted for the first few attempts. The voltage given by P at the null position is found by reference to a standard cell; or alternatively, the voltage V_P and the input voltage can be compared, using two resistors and a galvanometer, as in fig. 119, since it is the ratio of these voltages which is required rather than their absolute values.

Most computing elements other than integrators give an output voltage which is some desired function of an input voltage independent of time, and these can be tested and adjusted by the methods described for amplifiers, i.e. by measuring input and output voltages by means of voltmeters or potentiometers, or by the method of fig. 119. The remaining elements involve rotation of a shaft or some other mechanical motion for which methods of checking and adjusting are well known.

9.6 POWER SUPPLIES

In Chapter 5 it was indicated that d.c. amplifiers for use in analogue computers should be designed to be insensitive to variations of supply voltages, but in spite of this it is usually necessary, in all but the smallest and crudest computers, to use "stabilized" or "regulated" high-tension supplies if the equipment is to be operated from the public supply mains or an equivalent source. Any change in the voltage of such a source will cause some change in the h.t. voltage and hence some unwanted changes in the output voltages of the computing amplifiers. These changes of output voltage can be reduced by careful amplifier design, but it is generally economical to pay some attention to the stabilization of the h.t. supply rather than to rely entirely on making the amplifier insensitive.

Another advantage of stabilized h.t. supplies is that the level of ripple due to incomplete smoothing of the power-supply frequency and its harmonics is very small. Contamination of the output voltage of a computing element by ripple voltages of small amplitude is not, in itself, objectionable in some cases, but the system may contain elements such as phase-advancers or differentiators which give appreciable gain at ripple frequencies and there is then a danger that the amplified ripple may cause unsuspected overloads. In computers working with fast time scales (Section 10.2) the variations of output voltage of the computing elements may involve frequencies similar to the ripple frequencies and it is then necessary to keep the ripple level low so that the genuine variations can be properly observed.

A third and very important benefit which a stabilized h.t. supply source offers is a very low internal impedance. When an input voltage is applied to a computing element there is a re-distribution of currents within the element and, in general, there is a change in the total current drawn from the h.t. supply. This causes a change of h.t. voltage and this in turn may mean that the variation of output voltage due to the applied input is not precisely the same as if the h.t. voltage remained absolutely constant. This effect is not usually serious, because it is not difficult to check the overall performance of a computing element connected to its proper h.t. supply source. Suppose, however, that a second computing element is supplied from the same h.t. source. Then an input voltage applied to the first element may cause a variation in the output voltage of the second element; there may, in fact, be an unwanted coupling between the two elements via the internal impedance of the h.t. source, and there will be corresponding couplings between all the elements supplied from the same h.t. source. If there are appreciable gains in any of the elements, and if the h.t. source impedance is not very low the coupling may be sufficient to produce instability, but more usually the effect of the unwanted couplings is to upset the intended quantitative relations between the voltages in the various computing elements so that the computer gives incorrect answers. In principle the presence of coupling effects of this kind can be checked by earthing the input terminal of an element and observing its output voltage while an input voltage is applied to another element supplied from the same h.t. source. This is impracticable as a regular procedure, however, and it is therefore essential that the h.t. source impedance should be low enough to prevent appreciable coupling under all likely conditions. The actual value of tolerable impedance depends on the sensitivity of the computing elements to changes of h.t. voltage, on the type of problem being solved, the accuracy required, and also on the levels of computing voltages at various points in the computer, since a given spurious voltage at the output of a given element will be more serious when the legitimate output voltage is low. The internal impedance of the h.t. supply must be maintained at a low value for at least the whole range of frequencies occupied by the variables in the problem being solved, and preferably over the whole working frequency range of the computing elements.

It is fairly common practice in medium and large analogue computers to use more than one h.t. supply unit. This is done partly for convenience, in using a number of units of existing types rather than a single unit of special design, and it helps to avoid long runs of power-supply wiring. In addition, it greatly reduces unwanted couplings due to common h.t. source impedances. In some machines, including TRIDAC (Section 11.3), there is a common source of "raw" h.t. power, rectified and partially smoothed, which feeds a large number of separate stabilizing or regulating units, each of which feeds a small number of computing elements.

Most elements used in analogue computers need at least two h.t. supplies, at different voltages, and some designs need as many as four. The foregoing remarks on constancy of voltage, ripple content, and internal impedance, apply to all the h.t. supplies, although it cannot be assumed that characteristics which are acceptable for supply at one voltage are adequate or necessary for supplies at other voltages.

The h.t. stabilizer or regulator is normally of the thermionic type (Refs. 40 and 50) in which all the h.t. current flows between anode and cathode of a regulator valve. The output voltage is compared with that of some stable source, such as a gas-discharge tube or a battery, and the difference is amplified and applied to the grid of the regulator valve in such sense as to compensate for changes of output voltage.

Unwanted couplings between computing elements can occur not only in the "supply" side of the h.t. circuits but also in the earth-return leads. There is always a small voltage drop due to the return h.t. current flowing along an "earth" conductor back to the h.t. source. If this current changes when an input voltage is applied to the element the voltage drop will change slightly and this slight change may be added to the input and output voltages of other elements which use the same earth-return conductors. To reduce this type of coupling a heavy conductor is used, and some machines use copper bars of up to 0·5 square inch in cross section. In addition it is sometimes desirable to use more than one earth-return connection, and in some machines there are two distinct sets of "earth" conductors; the return h.t. currents are carried by one set of conductors, while the other set is used exclusively for the "earthy" input and output terminals of computing elements. The conductors of both sets are connected together at a single point. This arrangement is sound in principle, though in practice it is not always easy to distinguish precisely between h.t. return currents and currents associated with computing voltages, since all the computing currents are provided by the h.t. source. Some care is necessary in planning these multiple earth systems to ensure that the desired decoupling will be achieved.

Some attention must also be given to heater supplies for the valves in the computing elements. A given variation in the voltage of the public supply mains will give an immediate change of h.t. voltage and hence an immediate change of output voltage from a computing element, though the magnitude of the output change may be negligibly small. The same change in mains voltage will also produce a change of heater voltage, and this may well give a change of output voltage which is not negligibly small. The effect, however, will not be immediate because of the smoothing action of the thermal lags in the valve heaters and cathodes, and for this reason there is a tendency to overlook the effects of heater-voltage variation. In most computers, some degree of compensation is desirable, and it is common practice to achieve this by using a constant-voltage transformer. This transformer may be of the saturated-core type, or of one of the types in which the effective ratio is varied automatically, either by means of sliding tap-changing gear or by moving a short-circuited turn along a limb of the core so as to change the field distribution. This transformer may also be used to supply alternating current to the h.t. rectifiers, giving an additional measure of compensation against mains-voltage variations.

10

OTHER CLASSES OF SIMULATORS

The simulators which have been described in the earlier chapters mostly belong to a large and important class which have the following features in common:

1. They are "unity-time-scale" machines; i.e. if a particular event in the real dynamic system occupies a time interval T, then the analogous event in the simulator likewise occupies an interval T. Time constants and frequencies of oscillation have the same numerical values in the simulator as in the real system.
2. They are "d.c." "voltage" machines, which means that the analogous quantity in the simulator which corresponds to a variable in the real dynamic system is a voltage whose instantaneous value is proportional to the instantaneous value of the variable.
3. They are "explicit" machines, in the sense that each simple mathematical operation, particularly sign-reversal, addition and integration, is performed separately by one element of the simulator.

10.1 TIME SCALES

There are other important classes of simulator which do not share these features. One of the most important includes machines which have time scales different from the time scales of the systems being studied, so that an event which occupies an interval T in the real system occupies in the simulator an interval kT, where k is a time-scale factor which may have a value greater or less than unity. Suppose, for example, that in fig. 10 or fig. 11 the values of the integrator capacitances are all doubled. For a given charging current the rate of voltage rise on a capacitor is now only half the former value, and as there are no other time-conscious elements events proceed as before, but at half the rate. An event which previously occupied an interval T now occupies $2T$. If the capacitances were all halved instead of being doubled events would proceed twice as fast, and similarly for other changes of capacitance. There is a wide range of time scales in which the computation can be carried out, and depending on the circumstances there are advantages to be gained from computation rates either slower or faster than the one-to-one rate, and there are also some disadvantages in discarding the unity time scale.

The use of "slow" time scales, with a time-scale factor k greater than unity, is obviously valuable when the dynamic system includes rates of change so high that significant alteration in the value of some variable takes place in a time interval comparable with the lags in the simulator. Thus, suppose the dynamic system being studied includes a resonance at 50 c/s, and the simulator includes some computing element having a simple time lag with a time constant of 0·01 second. Such a lag would lead to errors if a real time scale were used, but if the computer were slowed down by using a time-scale factor of 10 the analogous resonance in the simulator would now occur at 5 c/s, and the 0·01 second time lag would be much less important. A larger factor than 10 would reduce the error still further.

As an extreme example of slow time scales it is possible to solve certain problems in geometrical optics by means of an analogue computer. For this purpose a particle may be imagined to move with a speed of, say, a few centimetres per second, its path being a straight line so long as the refractive index of the medium in which it moves is a constant, but changing direction in accordance with the usual laws at a reflecting or refracting surface. Such a dynamic system corresponds to an optical system, with the path of the particle representing a ray of light, and an analogue machine could be used to compute, one after another, the paths taken by different rays, so that the behaviour of the optical system could be examined. By the use of function generators curved surfaces, both spherical and aspherical, could be introduced, and with some additional complication some problems involving a medium with a continuously-varying refractive index, such as arise in electron optics, might be solved. Such a simulation would involve a time-scale factor of the order of 10^9, and would carry with it some implication of a corpuscular theory of light. For this purpose the corpuscular theory is quite acceptable for the optical case, and more obviously so for the electron optics.

10.2 FAST TIME SCALES; REPETITIVE SIMULATORS

Compared with slow time scales the use of fast time scales, with values of k less than unity has found wider application. An obvious field is in the simulation of systems which have inconveniently slow time scales, notably chemical plants, where events occupy minutes or hours rather than seconds. Another important application of fast time scales is to "repetitive" simulators (Refs. 1 & 51), in which the time scale is chosen so that the whole solution of a problem is completed in a period of, say, one fifth of a second or less. Such a simulator may be used to examine the response of a system to a step function, but instead of a single step the input voltage is a square wave, which corresponds to alternate positive and negative steps repeated indefinitely. The display of the response of the system to the negative steps is usually suppressed in some way, and the output voltage stimulated by the positive steps is fed to the Y plates of an oscilloscope which has a linear time base, operating at the same frequency as the square wave, connected to the X plates. Thus, provided the frequency of the wave and the afterglow of the oscilloscope screen are such as to prevent excessive flicker, there appears on the screen a stationary trace representing the step-function response of the system. A repetitive simulator has the advantage that the effects of changes in the parameters of the system are immediately obvious, and a large number of different combinations of parameters can be examined quickly. It has also the advantage that since the voltages in the simulator comprise only the fundamental and harmonics of the square-wave frequency with some sidebands there is no need to use direct-coupled amplifiers, and capacitance-coupled amplifiers may be used, provided they are designed to give negligible phase shifts at the relevant frequencies. This means that troubles due to drift and grid current are reduced, and there is also the possibility of operating from a smaller number of H.T. supply voltages than in the case of d.c. amplifiers.

At the beginning of each positive step there should be no residues of voltages from the response to the previous step, and to ensure this it is desirable to short-circuit the integrator capacitors by relay contacts which close during the negative step. This operation is unnecessary if it is certain that the natural decay of voltages leaves only a negligible residue, but the possibility of an appreciable voltage remaining at some intermediate point, even though the output itself is near zero, must not be overlooked. If the initial conditions for the problem are not all zero the initial-condition voltages are switched synchronously with the input square wave.

Whether the amplifiers be direct- or capacitance-coupled they have to operate at higher frequencies than the amplifiers for a unity-time-scale simulator, and this leads to stability difficulties if a very high loop gain is used with heavy feedback. Consequently, the amplifiers in a repetitive simulator have lower gains, and a value of around 2,000 is not unusual, compared with 50,000, say, for unity-time-scale amplifiers. This limits the accuracy, of course, but since the output trace is normally observed on a cathode-ray oscilloscope of normal design, which is not a very accurate amplitude indicator, this limitation is often unimportant. The individual computing element errors are often about 1·0%.

A disadvantage of the repetitive simulator is that it is restricted to the examination of the response of a system to brief stimuli. The usual stimulus is a step, which ideally occupies zero time, but other functions are possible, e.g. a triangular pulse, or a square-topped pulse, or a function which increases linearly from zero to some finite value and then remains constant at that value. It is essential, however, that the function involves changes of voltage only for some period which is less than half the square wave period and preferably much less if the consequent response is to be observed. The repetitive simulator is not very useful for problems which involve continuous input functions, such as the road vehicle problem of Section 4.3 or any problem in which the input contains continuous noise.

It is important to observe that the trace on the screen represents the response to a single occurrence of the stimulating function; the repetition is merely a convenient way of making the response easily and continuously visible. If the usual input square wave is replaced by a sine wave, sin ωt, of the same frequency, the output will not represent the frequency response of the system at frequency $\omega/2\pi$, or any simply-related frequency; it will be the response to a function which is identical with sin (ωtk) over the interval 0 to π, but which is zero for all other time, k being the time-scale factor which is less than unity for a fast-time-scale simulator.

The normal frequency response of the real system with continuous sinusoidal inputs can be measured by using the analogous network included in the simulator but omitting the square wave, synchronous switching, etc. The response of the simulator to a wave sin $(\omega t/k)$ will give the response of the real system to sin ωt.

10.3 INCLUSION OF COMPONENTS OF THE REAL SYSTEM

A disadvantage of all simulators which have time-scale factors different from unity is that parts of the real system cannot be inserted into the simulator loops. Consider, for example, a simulator for the study of a dynamic system which includes as a part of the system a feedback amplifier supplying a geared motor which drives a potentiometer, the potentiometer pick-off voltage being passed on to the next element. This combination needs an input voltage to operate it and gives another voltage as output, the output being equal to the input, say, in the steady state but lagging when the input varies. In the early stages of an investigation it would be sufficient to represent the whole combination by a simple RC lag (fig. 45), but at a later stage it might be necessary to introduce imperfections such as amplifier overload, motor and gearing friction and backlash, motor hysteresis, etc. Simulation of these effects is possible, but a simpler alternative is to connect the actual amplifier and motor combination into the simulator in place of the original RC circuit. This is, of course, not always practicable, but when it is it not only puts the imperfections into the simulator but also subjects the combination to realistic input variations.

The use of time-conscious parts of the real system as elements of the simulator is only possible, or at least only has any useful meaning, if the time scales of the real and simulated systems are equal.

10.4 NETWORK SIMULATORS

Besides the "explicit" simulators which have been exclusively discussed so far there is an interesting and sometimes useful class of "implicit" or "network" simulators, in which high-gain amplifiers are used with more complex input and feedback impedances in place of plain resistors and capacitors. By such means a single amplifier can be used to provide more than one elementary operation, so that economy of amplifiers may be achieved. As a simple example, consider the transfer function*

$$\frac{V_O}{V_1} = \frac{1}{1+pT}$$

* Strictly, the use of the "transfer function" requires that p should be redefined as the Laplace operator, but this would not change the form of the expressions.

In the explicit method of simulation this would first be re-written

$$V_O + V_O pT = V_1$$

or

$$V_O = \frac{V_1}{pT} - \frac{V_O}{pT} = \frac{1}{pT}(V_1 - V_O)$$

The corresponding computing circuit would need an integrator, a summing amplifier, and a sign-reversing amplifier, making a total of three high-gain amplifiers, arranged as in fig. 122. By contrast

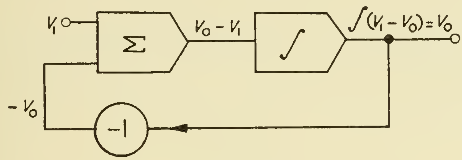

FIG. 122

with the simple arrangement of fig. 45, which consists only of a series resistor and a shunt capacitor, but has the same transfer function, fig. 122 seems very wasteful. However, this latter arrangement provides an output voltage from a low impedance, and both $+V_O$ and $-V_O$ are available; additional signals can be introduced merely by using more resistors at the inputs to any of the three amplifiers; initial conditions can be inserted without difficulty; and in some circumstances it is an advantage to have a resistive input impedance, independent of frequency, which this arrangement provides. By contrast, the input impedance of the simple arrangement

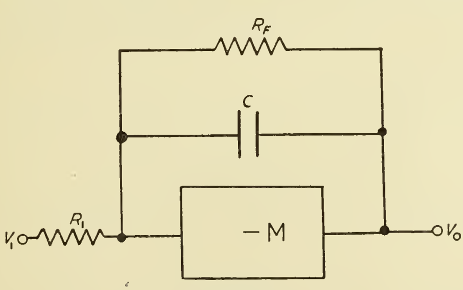

FIG. 123

varies with frequency, and the current drawn from the input source may vary with time, as for example when a step of voltage is applied.

An intermediate arrangement is shown in fig. 123, and the corresponding transfer function, assuming a very high amplifier gain, is

$$\frac{V_O}{V_1} = -\frac{R_F}{R_1}\frac{1}{(1+pT)} \quad \text{where } T = CR_F$$

This circuit has some of the advantages of the arrangement of fig. 122, and is one of a class represented by fig. 124, which shows an

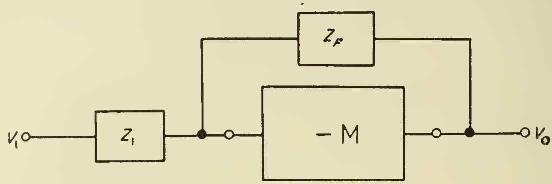

FIG. 124

input impedance Z_1 and a feedback impedance Z_F. The transfer function, assuming a very high amplifier gain is

$$\frac{V_O}{V_1} = -\frac{Z_F}{Z_1} \tag{45}$$

The simplest forms of impedance apart from single components are a resistor and capacitor in parallel or series. The values of impedance for these combinations are conveniently expressed in the forms:

$$\left.\begin{aligned} Z_P &= \frac{R_P}{1+pT_P} \\ Z_S &= \frac{(1+pT_S)R_S}{pT_S} \end{aligned}\right\} \tag{46}$$

as shown in fig. 125. Making use of these forms, the transfer functions obtainable by using any combination of single components or pairs of components can be derived directly. In a similar manner combinations of three or more components may be used, though the choice of a circuit to suit a particular transfer function is naturally less simple.

Four-terminal networks and other more complex arrangements can also be used in the input and feedback circuits in place of the usual two-terminal impedances. One such arrangement is shown in fig. 126, which shows a simple two-terminal network as the feedback impedance, but a four-terminal network in the form of a simple T

as the input circuit. If the value of M is so large that $1/M$ is negligible the overall response of this arrangement can easily be found by

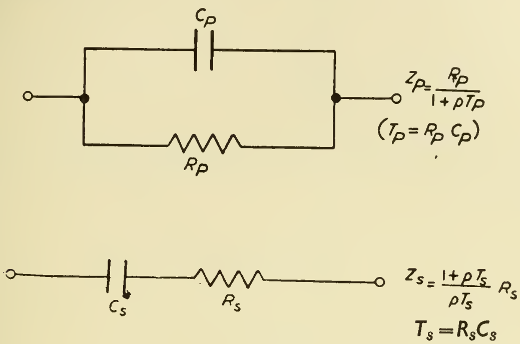

FIG. 125

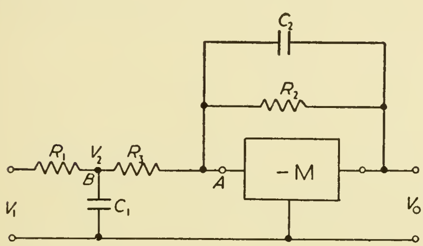

FIG. 126

making the assumption that the voltage at the amplifier input terminal A is zero. Ignoring grid current, the net current arriving at point A is zero, so that

$$\frac{V_2}{R_3}+\frac{Vo}{R_2}+VopC_2 = 0$$

or

$$V_2 = -Vo\left(\frac{R_3}{R_2}+pC_2R_3\right)$$

The net current arriving at B is also zero, so that

$$\frac{(V_1-V_2)}{R_1}-V_2pC_1-\frac{V_2}{R_3} = 0$$

or

$$\frac{V_1}{R_1}-V_2\left(\frac{1}{R_1}+\frac{1}{R_3}+pC_1\right) = 0,$$

and substituting for V_2 gives:

$$\frac{V_O}{V_1} = -\frac{R_2}{R_1+R_3} \cdot \frac{1}{(1+pT_1)(1+pT_2)} \tag{47}$$

where $$T_1 = \frac{C_1 R_1 R_3}{R_1+R_3}, \quad T_2 = R_2 C_2$$

This result may be compared with the corresponding expression for the arrangement of fig. 127, for which from equations (45) and (46) it is easily seen that

$$\frac{V_O}{V_1} = -\frac{R_2}{R_1} \cdot \frac{pT_1}{(1+pT_1)(1+pT_2)}$$

where $$T_1 = R_1 C_1, \quad T_2 = R_2 C_2$$

and although the denominator is of the same quadratic form as equation (47) the pT_1 term in the numerator is often undesirable.

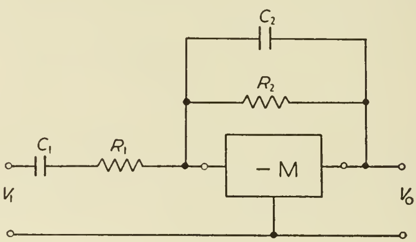

FIG. 127

The arrangement of fig. 126 is a convenient method of producing a second-order function of the low-pass type, but it is limited in its scope because the time constants T_1 and T_2 are necessarily real. It is sometimes required to produce an arrangement giving a response of the form

$$\frac{V_O}{V_1} = -K\frac{1}{1+2upT+p^2T^2} \tag{48}$$

which represents a simple second-order "resonance" of undamped natural frequency $1/2\pi T$ and damping ratio u. If u is greater than the "critical" value unity, which means there is no overshoot in the transient response, the denominator can be factorized into the same form as equation (47) and the response can be produced by the arrangement of fig. 126. If, however, u is less than unity the factors of $(1+2upT+p^2T^2)$ are complex, and the corresponding response cannot be reproduced by fig. 126.

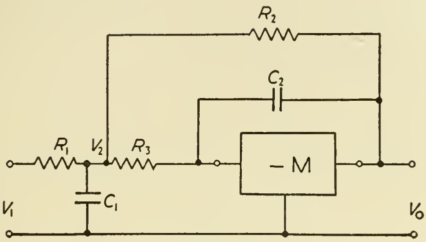

FIG. 128

A modification which allows production of responses with less than critical damping is shown in fig. 128. Assuming as before that M is very large,

$$\frac{V_2}{R_3} + V_O p C_2 = 0,$$

or

$$V_2 = -V_O p C_2 R_3$$

and

$$\frac{(V_1 - V_2)}{R_1} - V_2 p C_1 + \frac{V_O - V_2}{R_2} - \frac{V_2}{R_3} = 0$$

Elimination of V_2 gives:

$$\frac{V_O}{V_1} = -\frac{R_2}{R_1}\left\{\frac{1}{1 + pC_2\left(R_2 + R_3 + \dfrac{R_2R_3}{R_1}\right) + p^2C_1C_2R_2R_3}\right\} \tag{49}$$

Comparing this result with equation (48)

$$T^2 = C_1C_2R_2R_3; \quad 2uT = C_2\left(R_2 + R_3 + \frac{R_2R_3}{R_1}\right)$$

Let $R_2 = R_3 = R$, $R_1 = xR$, and $C_1 = yC_2 = yC$; then

$$T^2 = yC^2R^2, \text{ and } 2uT = CR\left(2 + \frac{1}{x}\right),$$

so that

$$u\sqrt{y} = 1 + \frac{1}{2x}$$

It is often undesirable to make x and y greater than about 20, and with this value $u = 0{\cdot}23$. This corresponds to a peak of 6·7 db

(2·24 :1 as voltage ratio) in the frequency response and an overshoot of about 44% in the transient response to a step function. Most practical servo systems have heavier damping than this, so their responses can be reproduced by the arrangement of fig. 128 if a second-order linear system is adequate.

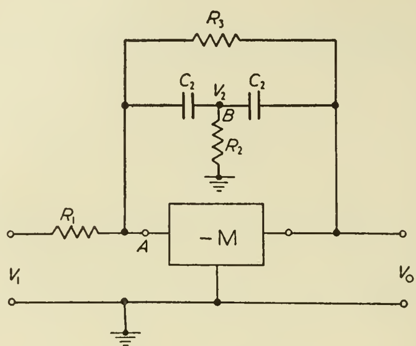

FIG. 129

Another arrangement which gives second-order responses with less than critical damping is shown in fig. 129, which has for its feedback circuit a four-terminal network of the bridged-T type. As before, the transfer function may be derived by equating to zero the net currents arriving at points A and B, i.e.

$$\frac{V_1}{R_1}+V_2pC_2+\frac{V_O}{R_3} = 0$$

and

$$-\frac{V_2}{R_2}+(V_O-V_2)pC_2-V_2pC_2 = 0$$

Elimination of V_2 gives

$$\frac{V_O}{V_1} = -\frac{R_3}{R_1}\left\{\frac{1+2pC_2R_2}{1+2pC_2R_2+p^2C_2{}^2R_2R_3}\right\} \tag{50}$$

and comparison with equation (48) shows that $T=C_2\sqrt{R_2R_3}$ and $u=\sqrt{\dfrac{R_2}{R_3}}$. If $R_3=20R_2$, $u=0{\cdot}225$, very nearly the same value as for equation (49) with $x=y=20$.

The denominator of equation (50) is simpler and rather more convenient for adjustment of values than the denominator of equation (49), but the numerator term $(1+2pC_2R_2)$ may not be needed, though it is sometimes useful as a method of introducing pure "phase advance" or derivative of the input voltage. The numerator term could be removed by using another network with a transfer function $\frac{1}{1+2pC_2R_2}$, connected in such a way that the two transfer functions multiplied without any loading or interaction effects. This would probably involve a buffer amplifier, and a more economical method is to modify the circuit of fig. 129 to that shown

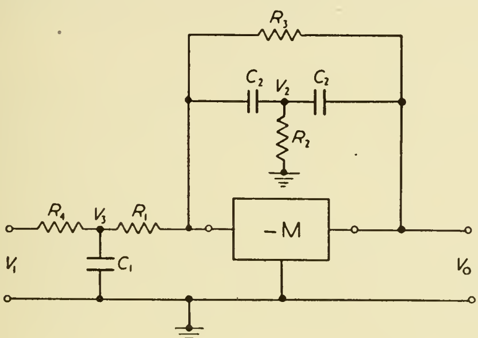

FIG. 130

in fig. 130, by adding a series resistor and shunt capacitor R_4C_1 in the input circuit. The relation between V_3 and V_1 is given by:

$$\frac{(V_1-V_3)}{R_4}-V_3pC_1-\frac{V_3}{R_1}=0$$

or

$$\frac{V_1}{R_4}=V_3\left(pC_1+\frac{1}{R_1}+\frac{1}{R_4}\right)$$

Writing R for R_1 and R_4 in parallel,

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_4}$$

and

$$\frac{V_3}{V_1}=\frac{R}{R_4}\left(\frac{1}{1+pC_1R}\right)=\frac{R_1}{(R_1+R_4)}\left(\frac{1}{1+pC_1R}\right)$$

Now equation (50) shows that, for fig. 130,

$$\frac{V_O}{V_3}=-\frac{R_3}{R_1}\left(\frac{1+2pC_2R_2}{1+2pC_2R_2+p^2C_2{}^2R_2R_3}\right)$$

so that

$$\frac{V_O}{V_1} = -\left(\frac{R_3}{R_1+R_4}\right)\left(\frac{1+2pC_2R_2}{1+pC_1R}\right)\left(\frac{1}{1+2pC_2R_2+p^2C_2{}^2R_2R_3}\right) \quad (51)$$

and a pure second-order response may be obtained by setting $pC_1R = 2pC_2R_2$. Alternatively, of course, the terms $(1+2pC_2R_2)$ and $(1+pC_1R)$ may be retained to produce more complex responses, a particular application being the reproduction of an oscillatory response, represented by the quadratic term, combined with an "impure" phase advance represented by the two linear terms. For this purpose the ratio of time constants $2C_2R_2/C_1R$ would commonly lie between about 3·0 and 20.

There is some scope for ingenuity and skill in devising economical amplifier-plus-network combinations to satisfy particular requirements, and with some practice and experience it is often possible to forecast the nature of the change in the transfer function resulting from a change in the circuit without the labour of solving the equations. As a simple example, suppose it is required to modify the arrangement of fig. 128 whose transfer function is given in equation (49), to give a transfer function of the form

$$\frac{V_O}{V_1} = -K\frac{1+pT_1}{1+2upT+p^2T^2} \quad (52)$$

which is similar in form to equation (49), except for the additional numerator term. There is no guarantee that this is possible, but the new expression resembles the original; and there are five component values available for adjustment in the circuit compared with only three parameters T, T_1 and u, in the transfer function, so there is a reasonable hope of success.

Comparison of equations (52) and (49) shows that (52) requires an additional term V_1pT_1, and a means is required of adding such a term in the equations expressing the network currents. One method would be to use another capacitor in parallel with R_1 (fig. 128), giving an extra term $(V_1 - V_2)pC$ in the current equations. Bearing in mind the desire for economy, however, it is worth examining the possibility of using the existing capacitor C_1 in this manner, giving the circuit shown in fig. 131. The transfer function of this arrangement can be derived from equation (49), and it is convenient to take two steps. First, it may be imagined that C_1 is removed, so that the coefficient of p^2 in equation (49) is now zero. Second, a capacitor

C_3 is added in parallel with R_1 and hence in the transfer function R_1 is replaced by the impedance of C_3 and R_1 in parallel, which is

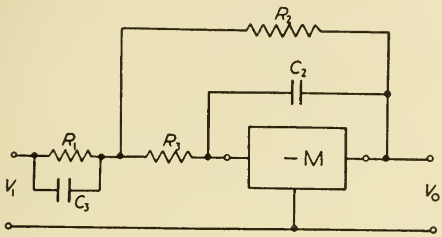

FIG. 131

$R_1/(1+pC_3R_1)$. This gives a numerator term $(1+pC_3R_1)$ and introduces a term $p^2C_2C_3R_2R_3$ into the denominator, the complete transfer function being now:

$$\frac{V_0}{V_1} = -\frac{R_2}{R_1} \cdot \frac{1+pC_3R_1}{1+pC_2\left(R_2+R_3+\dfrac{R_2R_3}{R_1}\right)+p^2C_2C_3R_2R_3}$$

This is of the desired form, and is also of the same form as equation (50) but without the restriction that the coefficients of p in numerator and denominator are equal.

All the amplifier-plus-network combinations described so far make use of one amplifier only and give transfer functions substantially independent of the gain, provided the gain is very high. Arrangements using more than one amplifier are possible, and one such circuit which has some attractive features has been described by

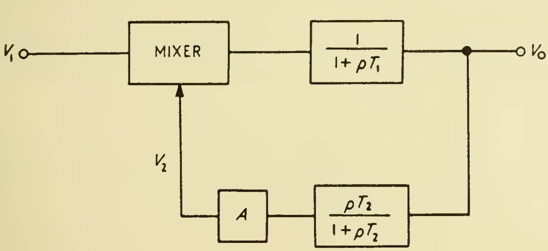

FIG. 132

D. V. BLAKE (Ref. 4). The circuit is shown in fig. 132; it makes use of a "mixer", which gives an output equal to the difference between two input voltages, and an amplifier of gain A, together with networks to give the two transfer functions $1/(1+pT_1)$ and $pT_2/(1+pT_2)$ (see figs. 45 and 43). Then

$$V_O = \left(\frac{1}{1+pT_1}\right)(V_1 - V_2)$$

$$V_2 = A\frac{pT_2}{1+pT_2}V_O$$

whence

$$\frac{V_O}{V_1} = \frac{1+pT_2}{1+p\{T_1+(1+A)T_2\}+p^2T_1T_2}$$

This represents a second-order resonance of undamped frequency $1/2\pi\sqrt{T_1T_2}$ and damping ratio $\frac{1}{2}\{T_1+(1+A)T_2\}/\sqrt{T_1T_2}$; or if $T_1=T_2$ the damping ratio is $\frac{1}{2}(2+A)$. Thus, a variation of A from zero to -2 gives damping ratios between unity (critical damping) and zero although when the damping ratio is small it varies rather rapidly with the value of A, and special precautions may be needed to maintain the gain sufficiently constant. Besides the wide range of damping ratios available there is the advantage that the damping can be varied without varying the coefficient of p^2. Negative damping ratios, giving continuous oscillation, can easily be achieved if A is made negative with $|A|>2$ when $T_2=T_1$. If T_2 is greater than T_1 a smaller value of $|A|$ will give negative damping.

The numerator term $(1+pT_2)$ may be unwanted in some applications though there is the advantage that the time constant T_2 is not automatically fixed by the frequency and damping of the denominator as in some other circuits. The term could be removed by adding a resistor and capacitor in front of the mixer, to give an additional term $1/(1+pT_2)$.

A derived form of BLAKE's circuit, using one high-gain amplifier and one sign-reversing amplifier, is shown in fig. 133. The transfer function is

$$\frac{V_O}{V_1} = -\frac{R_2}{R_1}\left\{\frac{1+pC_3R_3}{1+p(C_2R_2+C_3R_3-C_3R_2)+p^2C_2C_3R_2R_3}\right\} \tag{53}$$

or writing $C_3=C$, $C_2=xC$, $R_3=R$, $R_2=yR$, $CR=T$;

$$\frac{V_O}{V_1} = -\frac{R_2}{R_1}\left\{\frac{1+pT}{1+pT(xy+1-y)+p^2T^2xy}\right\}$$

A wide range of damping ratios can be achieved by choosing suitable values for x and y. An interesting property is that if

$x = y = 1$ the damping is half-critical, whatever the values of C_3 and R_3. If $x = (1 - 1/y)$ the damping is zero, and any smaller value of x gives an unstable system. Accurate control of the values of x and y is necessary if a particular damping ratio is to be realized precisely.

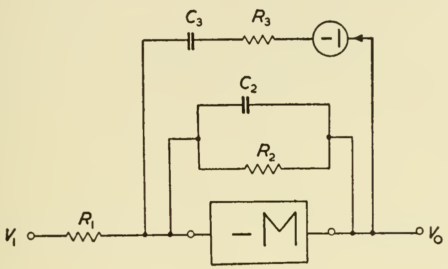

Fig. 133

Also, although it does not appear in an obvious way in the equations, the gain of the sign-reversing amplifier must be accurately controlled at the proper value. A disadvantage of this circuit compared with Blake's arrangement is that the damping ratio cannot be varied in any easy way without varying the frequency also. If this facility is desired it can be arranged by changing the values of the feedback or input resistors of the sign-reversing amplifier so as to give a gain of value $-N$ instead of -1. The transfer function then becomes:

$$\frac{V_O}{V_1} = -\frac{R_2}{R_1}\left\{\frac{1 + pT}{1 + pT(xy + 1 - Ny) + p^2T^2xy}\right\}$$

and besides the facility of easy adjustment of damping it is now possible to fix independently the values of frequency and damping of the denominator and the coefficient of p in the numerator. An advantage of arrangements of the type shown in fig. 133 compared with Blake's original arrangement is that the output impedance of the circuit is very low, provided the resistors have reasonable values, so that V_O can be fed into another circuit without the need for a buffer amplifier.

If a capacitor C_1 is added in parallel with R_1 in fig. 133 an extra factor $(1 + pC_1R_1)$ is introduced into the numerator, so that

$$\frac{V_O}{V_1} = -\frac{R_2}{V_1}\left\{\frac{(1 + pC_1R_1)(1 + pC_3R_3)}{1 + p(C_2R_2 + C_3R_3 - C_3R_2) + p^2C_2C_3R_2R_3}\right\}$$

when the sign-reversing amplifier has unity gain. Again, the coefficient of p in the numerator of equation (53) may be removed if desired by adding a series resistor and shunt capacitor in the input circuit of fig. 133, just as fig. 129 was modified to give fig. 130 (equations 50 and 51).

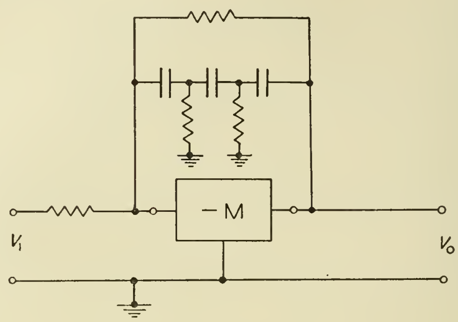

Fig. 134*a*

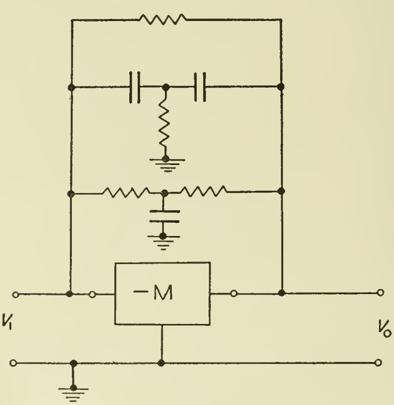

Fig. 134*b*

Transfer functions representing resonances with light damping can be achieved with a single amplifier if either a resistance-capacitance ladder or a pair of resistance-capacitance T-networks in parallel is used as feedback networks, as shown in fig. 134. The first of these circuits is basically the same as the ordinary "phase-shift" type R-C oscillator, but the loop gain is reduced below the value necessary for sustained oscillations. The second circuit is similar, but in this case the corresponding oscillator is of the "Wien-bridge" type, the double-T arrangement being a transformed version of the standard bridge circuit. These arrangements can give satisfactory performance in certain cases once they are set up to a given frequency and damping, but the expressions for the transfer functions are rather complex, so that adjustment is not easy. For use in a flexible analogue computer these disadvantages are generally sufficient to outweigh the economy in amplifiers compared with fig. 132 or fig. 133.

10.5 A.C. SIMULATORS

There is another class of analogue computing machines which use voltages as the analogue quantities, but which use "a.c." rather than "d.c." (Ref. 44). In such machines a carrier wave is used, and the instantaneous amplitude of a variable in the real system is proportional to the instantaneous amplitude of the envelope of the carrier. Thus, a variable in the real system represented by $f(t)$ would appear in the simulator as $f(t) \,.\, k \sin (2\pi f_c t)$, where k is a scale factor and f_c is the carrier frequency. The various modulated carriers can be added, subtracted, and multiplied by constants, in capacitance-coupled amplifiers, so that drift and grid-current troubles are unimportant and errors can be kept small. For integration, differentiation, smoothing, and any operation which involves time derivatives or integrals of the envelope of the carrier it is not possible to use the methods which are satisfactory for d.c. machines because these methods operate on the instantaneous value of the input voltage and so give the derivative, etc., of the carrier and not of its envelope. In communication engineering there is a technique of low-pass to band-pass transformation (Ref. 14) by which the characteristics and circuit arrangement of a band-pass device, say a band-pass filter, can be deduced from the corresponding low-pass device. The use of an alternating carrier instead of "d.c." in an analogue computing machine is effectively a low-pass to band-pass transformation, and using the methods of communication engineering it is possible to derive circuits corresponding to the integrator and other d.c. computing elements. These a.c. circuits will contain, for example, parallel-tuned combinations of inductance and capacitance in place of single capacitors in the d.c. circuits, and although this gives arrangements which will, in theory, provide the required operations on the carrier it is found in practice that it is difficult to keep the anti-resonant frequency of the tuned combinations exactly equal to the carrier frequency and also to realize sufficiently low losses in the components. Some practical work has been done on these lines and also with resistance-capacitance networks equivalent to tuned circuits (Refs. 45 and 46) but the more usual procedure is to demodulate the alternating voltage, perform the desired operation on the demodulated d.c. signal, and remodulate. This procedure introduces d.c. errors during the operation on the demodulated signal and detracts from the benefits of using a carrier wave. Furthermore, in order to preserve the signs of the analogue quantities the demodulators must be of the phase-sensitive type, and the presence of any quadrature component in the demodulating carrier gives an additional error. The attractions of this type of simulator have diminished since the development of satisfactory drift-stabilized d.c. amplifiers.

There is another quite distinct class of analogue computers, using alternating voltages with "synchros", of the types used for data transmission, as the computing elements. Such computers are attractive for some applications, since they enable addition, subtraction, and especially resolution, to be performed conveniently and accurately. However, they suffer from the same disadvantage as other a.c. systems when operations involving integration or differentiation are required, and for automatic operation some of the synchros must be driven by servo motors, which may introduce undesirable lags.

11

SOME EXISTING COMPUTERS

Many analogue computers have been designed and built in Britain and elsewhere, and descriptions of a few of these machines will be given in this Chapter.

One of the first complete electronic computers to be publicly demonstrated in Britain was a "five-integrator" analyzer, built by Gait and Allen at the Royal Aircraft Establishment and shown at the Physical Society Exhibition in 1948 (Ref. 30). On the same occasion a variable-mark-space multiplier was also shown.

11.1 REPETITIVE COMPUTERS

In 1949 a demonstration was given of the Electronic Simulator D.A., Mk. 1, designed and built in Britain by the Sperry Gyroscope Company (fig. 135) (Ref. 47). This is a "repetitive" machine of the d.c. analogue type, with a generator providing either square waves or saw-tooth waves, so that the response of a system to either a step function or a "ramp" function (for which $V=0$, $t<0$; $V=kt$, $t>0$) can be examined. The repetition frequency can be varied between 0·5 and 50 c/s. The machine includes five adding units, each of which comprises a single cathode-follower with four resistors connected to the grid terminal so that up to four quantities can be added together. This device is not a summing amplifier, and the coefficient by which each of the input quantities is multiplied depends on the values of all the input resistors (assuming they are all in use) so that adjustment of the coefficients is not easy. However, if the resistors are all equal and are all fed from low-impedance sources (or earthed if not in use) the coefficients are also equal, and the need for variations in the coefficient values can usually be satisfied by adjusting the gains of the units providing the input voltages to the adder. The machine also has ten units which are called integrators for convenience but which are actually capable of a variety of operations. Associated with these units are a number of R-C units, each containing two resistors, adjustable in steps of 1,000 ohms from 1,000 ohms to 7·0 megohms, and two capacitors, one adjustable from 0·001 μF to 7·0 μF and the other from 0·001 μF to 1·0 μF. These four components can be connected singly, or in series or parallel combinations, and they are used to provide the input and feedback impedances for the amplifiers. Thus a wide variety of transfer functions, of the types indicated in fig. 124 and equation (45) are available, and changes of value and of circuit arrangement can be made quickly and easily. The amplifiers in these "integrator" units are directly-coupled and have a gain of 2,500. Long-tailed-pair amplifying stages (Ref. 11) are used, with two cathode-follower output stages, so that two output voltages are available, of equal magnitude and opposite sign, both from low-impedance sources. There is thus no need for sign-reversing amplifiers, and circuits of the type shown in fig. 133 can be realized with a single amplifier.

The voltage representing the quantity which is to be examined is fed to the Y-deflection plates of an oscilloscope, and the X-deflection plates are supplied with a recurrent linear sweep voltage synchronized with the square-wave or saw-tooth wave which supplies the input voltage to the simulator. A stationary curve representing the response of the system appears on the screen of the oscilloscope tube, and a tracing of the curve can be made directly on a strip of matt-surfaced semi-transparent film which is carried by guides above and below the screen. Timing pulses at intervals of 0·5 millisecond to 50 millisecond are provided and can be displayed on the screen.

The machine is self-contained, with its own stabilized power supplies, and is mounted in a console type cabinet with the oscilloscope tube in a central position.

Another repetitive simulator of the d.c. analogue type has been constructed at the National Physical Laboratory and described by Blake (Ref. 4). The machine includes a number of time delays, each comprising a series resistor and a shunt capacitor (fig. 56), with buffer amplifiers. Adjustments of values are provided which give a range of time constants of 10,000:1. Directly-coupled amplifiers are provided, having stabilized gain and a high cut-off frequency; the heater supply as well as the h.t. supply of these amplifiers is stabilized. A number of units called "mixers" give output voltages equal to the difference between two input voltages, and they can also be used, with suitable inputs, as buffer amplifiers, with or without a

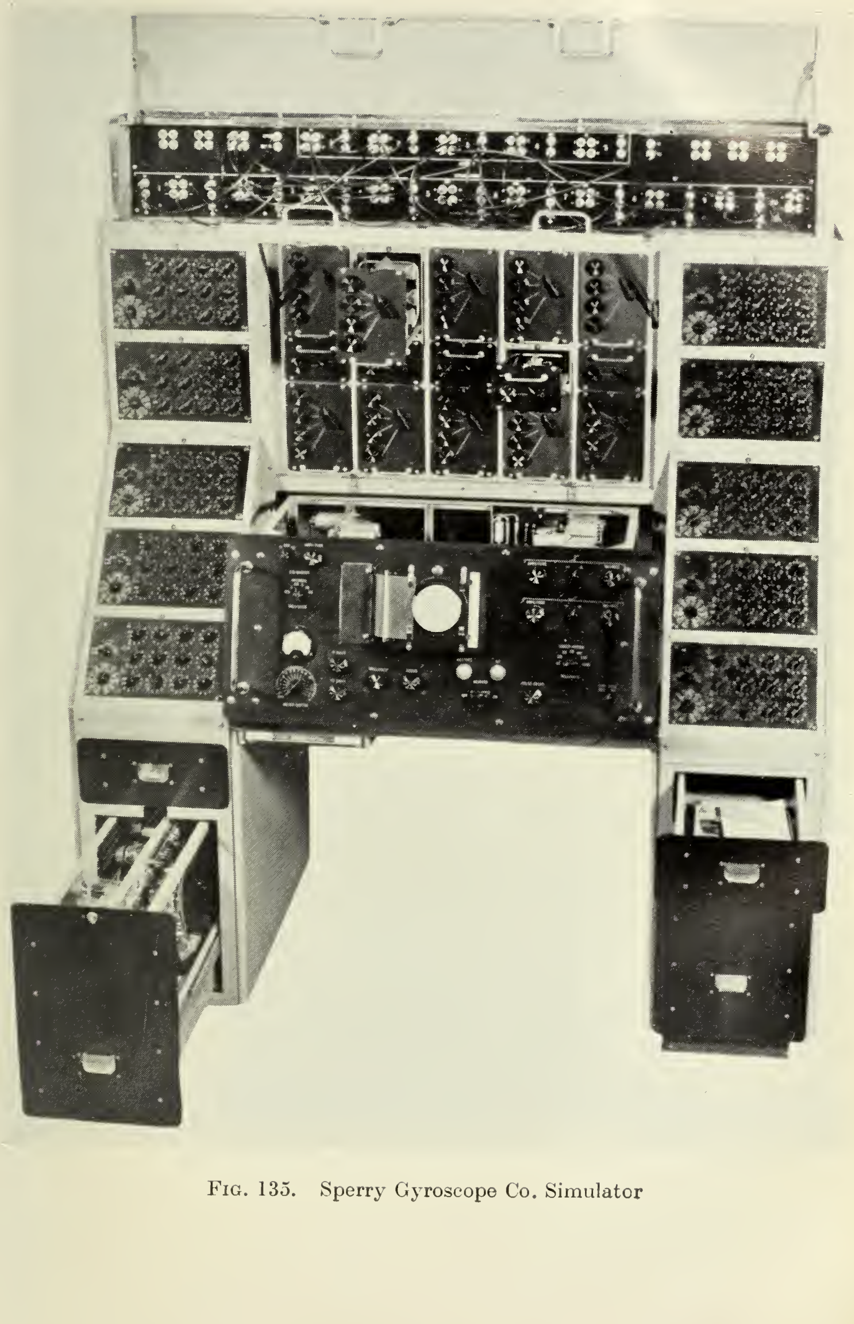

Fig. 135. Sperry Gyroscope Co. Simulator

reversal of sign between input and output. The integrators are of the conventional type using a balanced amplifier with a gain of about 2,000 and time constants between 0·5 and 0·05 second are available Circuits of the "phase-advance" type (fig. 136) are provided, with provision for varying the time-constant of phase advance without changing the attenuation. Each phase-advance circuit is followed by a d.c. amplifier to give zero overall attenuation at low frequencies and to provide a low output impedance.

$$V_0 = \alpha \frac{1 = pT}{1 \times \alpha pT} V_1$$

$$\alpha = \frac{R_2}{R_1 + R_2}$$

$$T = CR_1$$

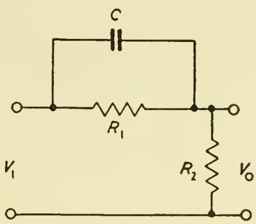

FIG. 136. Phase-Advance Circuit

This simulator has been designed primarily for the study of process control and the behaviour of chemical plants fitted with various types of automatic control. For this purpose two additional units have been installed, *viz.* the pure time delay or distance/velocity lag, and the on-off controller. The pure time delay is used to represent, for example, the flow of water of varying temperature along a pipe. If the rate of flow is constant, and if heat losses along the pipe are neglected, the temperature of the water at the output end will vary in time in exactly the same manner as at the input end,

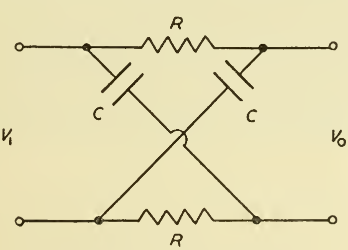

FIG. 137. Simple Lattice

but with a displacement in time corresponding to the time of flow from one end to the other. This time displacement is, of course, quite independent of the rate at which the temperature changes, and it cannot be simulated by any single simple network. An electrical network to give a constant time delay, independent of the frequency content or rate of variation of the input voltage, must have a transfer function of the form e^{-pT}, which leads to the requirement of zero attenuation and a phase-shift proportional to frequency. The simple lattice network shown in fig. 137 has a transfer function

$$\frac{V_O}{V_1} = \frac{1-pT}{1+pT} \quad \text{where } T = RC,$$

and a frequency response

$$\frac{V_O(j\omega)}{V_1(j\omega)} = \frac{1-j\omega T}{1+j\omega T}$$

and it is easily shown that the attenuation at all frequencies is zero, while the phase shift is very nearly proportional to frequency if ωT is small. Similar responses can be obtained with other "all-pass" networks (Refs. 14, 48). In the N.P.L. simulator a set of 15 networks is provided, with buffer amplifiers between successive networks, and each network provides a phase lag, at the maximum working frequency, of 20°. The networks can be used in groups of five, each group giving a lag of 100°, and if all the time constants are made equal the total lag is 300°. The departure from linearity of the phase-shift/frequency relation is about one per cent.

The on-off controller simulates a device in common use in process control systems. It has only three available output voltages, *viz.* zero and $\pm V_k$. Whenever the input voltage V_1 lies within a range $\pm V_d$, called the "dead zone", the output voltage is zero; when the magnitude of the input voltage exceeds V_d the output voltage is V_k, the sign being the same as that of V_1. In other words the output voltage V_O is such that

$$V_O = -V_k,\ V_1 < V_d$$
$$V_O = 0,\ -V_d < V_1 < V_d$$
$$V_O = V_k,\ V_1 > V_d$$

The value of V_d is adjustable, and also feedback can be provided from the output to the input so that the value of V_d is changed when the output voltage V_k appears. Thus, if V_1 first of all increases steadily and then decreases the effective value of V_d will be different during increase and decrease, so that a hysteresis effect can be introduced.

More recently, a convenient and versatile computer of the repetitive

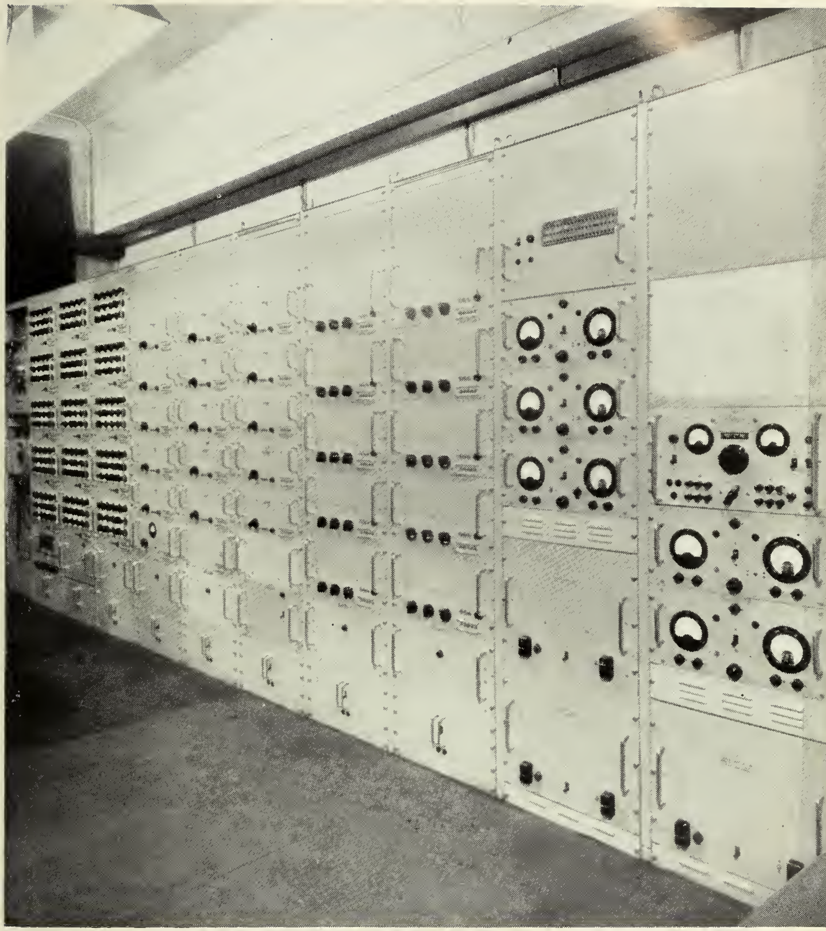

FIG. 138. GEPUS – Main Computing Section

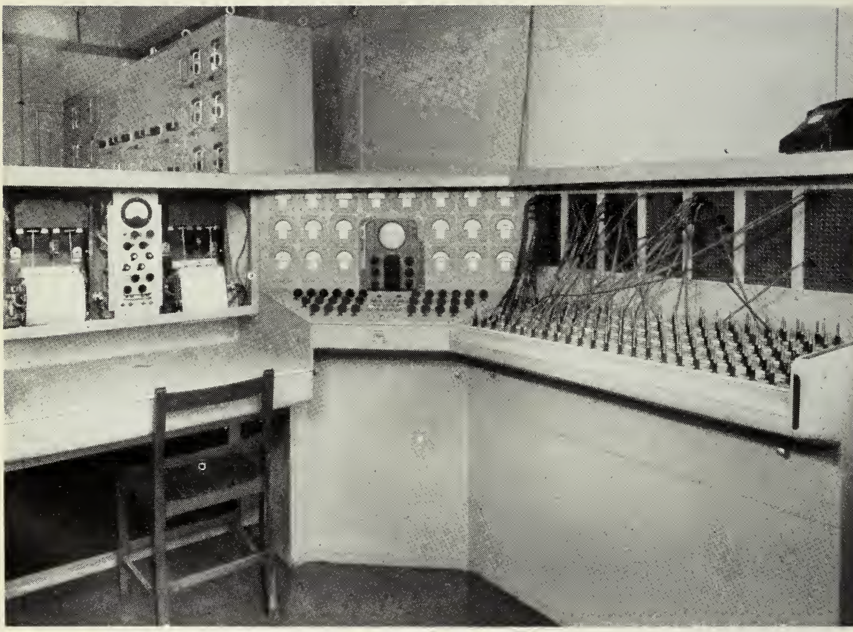

FIG. 139. GEPUS – Control Desk

type has been produced on a commercial scale by Short Bros. and Harland (Belfast). This is a self-contained machine with a layout similar to the Sperry machine, and with a number of attractive features, including an automatic zero check at the end of each sweep and means for rapid interconnection of units. It is primarily intended for the solution of stability problems of the types which arise in the study of aircraft vibration and flutter, but can tackle other kinds of problem, and has provision for some non-linear operations.

11.2 GEPUS

A number of analogue computers have been built at the Royal Aircraft Establishment, and of these the two of greatest general interest are "GEPUS" and "TRIDAC". GEPUS, the general purpose simulator (figs. 138, 139) was designed as a flexible and versatile machine of the d.c. analogue type, capable of solving with moderate accuracy the equations arising in fairly complex dynamic systems. SQUIRES was mainly responsible for the design, and the engineering and manufacture were by the Plessey Co. (Ilford). The computing elements and power supplies occupy 15 standard racks (6′ 6″ × 19″). There are 15 summing amplifiers, 16 integrators, 12 multipliers, 12 curve followers, and 30 units called "combined" units. The d.c. amplifiers are similar to the type shown in fig. 42, and have a gain of about 50,000. Each summing amplifier has five input resistors, and each resistor comprises two switched decades and a variable resistor, giving fine adjustment over a range of 80 kilohms to 10 megohms. Lower resistance values can also be obtained but are not usually used because of the danger of errors due to loading of the preceding element. The feedback resistor is normally fixed at 1·0 megohm. The integrators are of the Miller type using polystyrene capacitors, and the time constants of integration can be set to any of ten values between 0·01 and 10 seconds. Intermediate values can also be obtained but have not been found of great usefulness, since the effective time constants can be adjusted by varying the gains of preceding or following elements. The integrators can be connected as differentiators by the turn of a switch, but for reasons already explained (Section 6.7) differentiation is avoided whenever possible.

The multipliers are of the variable-mark-space type described earlier (Section 7.1), and they are normally used not only for straightforward multiplication but also for the generation of trigonometrical functions by computing the first two or three terms of series; this procedure is particularly useful when the angles are a little too large to allow the usual "small-angle" approximations. The curve followers use photocells and cathode-ray tubes as described in Section 7.4 and the masks representing the curve to be followed are photographed on lantern plates ($3\frac{1}{4}'' \times 3\frac{1}{4}''$), although if poorer accuracy can be tolerated a mask cut from stiff paper may be used. The frame carrying the mask can be adjusted in angular position, to set the axes of the curve parallel to the deflection directions of the c.r. tube, and also in two perpendicular directions laterally so that a particular point on the mask can be set opposite the centre of the screen.

The "combined units" are high-gain amplifiers with a choice of a few fixed values of input and feedback resistors, giving gains of 10, 1·0, 0·1 and 0·01. They can be used for sign-reversing, for inserting gain or attenuation, and for inserting initial conditions.

The machine includes altogether about 90 high-gain amplifiers without automatic drift correction, and zero-setting by the usual method of short-circuiting the input and turning a knob by hand to give zero output would not only be laborious, but also rather ineffective, since there would be time for the first amplifier to drift appreciably before the last one was adjusted and the actual computation begun. To overcome this difficulty Squires has devised an automatic zero-checking and setting system. A standard uniselector is arranged so that on its first step away from its "home" position the input terminals to one particular amplifier are short-circuited, and the output terminals are connected to a sensitive polarized relay having changeover contacts, adjusted so that the moving contact is midway between the two fixed contacts when no current flows in the winding. Contact between the moving contact and one of the fixed contacts occurs when the output voltage of the amplifier lies outside the range $\pm 0{\cdot}1$ volt, and the closing of the contact starts a small motor which turns the zero-setting potentiometer in the appropriate direction so that the error is reduced. The voltage at which the relay releases is less than 0·1 volt, so that although correction is not applied until the error is 0·1 volt the error when the relay releases is somewhat less than this value, and the final error is a little less again because a further small reduction takes place while the motor is slowing down. The time taken for the motor to slow down limits the speed at which it can be run, and at this rather slow speed large errors would take an inconveniently long time to correct. A second, less sensitive, relay is therefore provided and if the error is large enough this relay operates and causes the motor to run faster. During the zero-setting process the input and feedback resistors which are to be used for the computation are switched out of circuit and temporarily replaced by a pair of resistors which give an overall gain of about 100, so that the zero-setting error of $\pm 0{\cdot}1$ volt corresponds to about one millivolt at unity again.

When the zero-setting is completed and the sensitive relay released the uniselector is made to step to its second position, in which it short-circuits the input terminals of a second amplifier, connects the relays across its output terminals, and the zero-setting procedure starts up as before. This is repeated automatically for all the amplifiers and also for the integrators and multipliers, the integrator capacitors being temporarily replaced by resistors.

If any unit shows an output, with the input terminals short-circuited, of less than the 0·1 volt necessary to operate the sensitive relay the uniselector automatically steps on to the next unit. To reduce the total time needed to set all the zeros three uniselectors are used, operating independently and each associated with one group of amplifiers or multipliers. Each uniselector has its own pair of voltage-sensitive relays, with other relays for the automatic stepping; and each amplifier or other element has its own motor-driven zero-setting potentiometer and also a second pre-set potentiometer which is adjusted when a valve or component is changed so that the normal position of the motor-driven potentiometer is roughly central. The zero-setting procedure is so quick that it can be carried out before every run on the simulator; when the machine is warm and the zeros have been correctly set a few times, only a small number of the elements drift sufficiently between runs to operate the voltage relays and the whole procedure occupies 10 to 20 seconds.

On one bank of contacts of each uniselector each contact (except the "home" contact) is connected to a small lamp, so that there is one lamp for each computing element in the machine. If a fault occurs in any of the elements there is a high probability that the d.c. conditions will be disturbed to such an extent that the motor-driven potentiometer will have insufficient range of adjustment to give "zero out for zero in". When this happens the uniselector does not step on to the next element, and the lamp corresponding to the faulty element remains alight, giving obvious indication of the fault. There is provision for an audible alarm to be given also. From this fault condition the zero-setting mechanism can be set in motion again by the manual operation of a re-start button, which steps the uniselector on to the next element. In the home position of each uniselector another indicator lamp is connected, and all these lamps light when the uniselectors return to their home positions, indicating that all the elements have been tested. A simple electrical interlock provides that computation cannot begin until all these indicator lamps are alight.

Besides indicating faults so serious that zero-setting is impossible this equipment also sometimes reveals incipient faults when one particular unit needs its zero adjustment to be made more frequently than usual.

The input terminals of all the computing elements are connected to a set of telephone-type plugs, and the output terminals are connected to a set of jacks. The plugs and jacks are all mounted on a control desk (fig. 139) and for each pair of output terminals there are several jacks connected in parallel so that the output of one element can easily be connected to the inputs of several other elements. Adjacent to the jacks for each element is mounted the lamp associated with the zero-setting device.

Mounted on the control desk are four high-speed pen recorders, a cathode-ray oscilloscope, potentiometers for setting initial conditions, and a number of meters. The recorders, oscilloscope and meters are all connected to plugs alongside the input-terminals plugs, so that they can be connected as desired to the output terminals of any of the computing elements. Also mounted on the desk are the indicator lamps which show when the zero-setting has been completed, the re-start buttons for use when a uniselector has stopped at a faulty element, and also a master "start" button for the whole simulator.

Every computing element is fitted with a "start" relay which in the unoperated condition earths the ends of the input resistors remote from the first grid, and shunts the integrator capacitors with resistors so that they are completely discharged. When the "start" button is pressed, all these relays are operated simultaneously, and the computation begins. The step function or other input disturbance is also switched on at the same time, or with a very small time delay, if desired, to ensure that all the relays have had time to switch their respective computing elements into circuit. If the pen recorders are to be used the paper-drive motors are automatically started at the same time. A "stop" button reverses the action of the start button, disconnecting the computing elements, discharging the integrator capacitors, and stopping the recorder motors.

The total power consumption of GEPUS is about 7 kilowatts, supplied from the a.c. mains via an automatic regulating transformer of the sliding-contact type. This corrects slow changes of mains voltage to within about one per cent for heater supplies, and conventional thermionic stabilizers are provided in addition for the high-tension supplies. A valve-ageing panel is fitted and is filled with valves of the types used in the machine. These valves are switched on whenever the machine itself is switched on, and whenever a valve is taken from this panel to replace a faulty valve in the machine a new valve is put in its place on the panel. The anode currents of the valves on the panel are checked occasionally and faulty valves are replaced, and the number of valves is so chosen that every valve gets at least a few dozen hours running before going into service. With this simple procedure it is found so far that the 850 valves in the machine fail at a rate of roughly one per week.

GEPUS has been working for several years, and has proved reliable and versatile; on occasion two entirely separate problems have been set up at the same time. Little time has been lost through faults because, except when all the elements of a particular type are in use, a faulty element can be taken out of circuit and replaced by a good one, and computation can proceed while the faulty element is being repaired elsewhere. The accuracy has been found to be a little better than expected, largely because of the improving skill of the operator.

11.3 TRIDAC

TRIDAC (figs. 140 and 141) is the largest computing machine so far built in Britain, and among the largest in the world. Its name is derived from "three-dimensional analogue computer", and its design has been based on the need for the solution of problems of aircraft flight in three dimensions. TRIDAC is a d.c. analogue machine and includes about 650 high-gain amplifiers, all of them with automatic drift correctors, some of the chopper-relay type and some of the magnetic modulator type. There are also a number of mechanical computing devices, driven by nine high-speed hydraulic motors which absorb about 400 mechanical horsepower under peak load

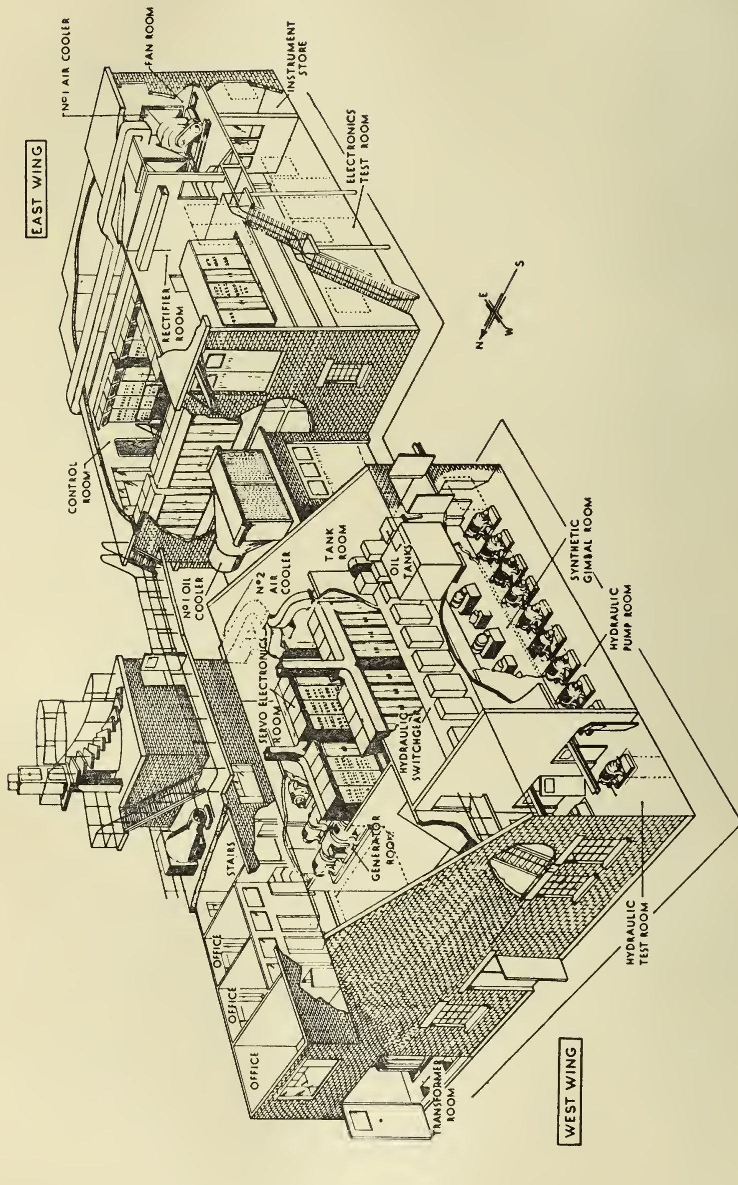

Fig. 140. TRIDAC

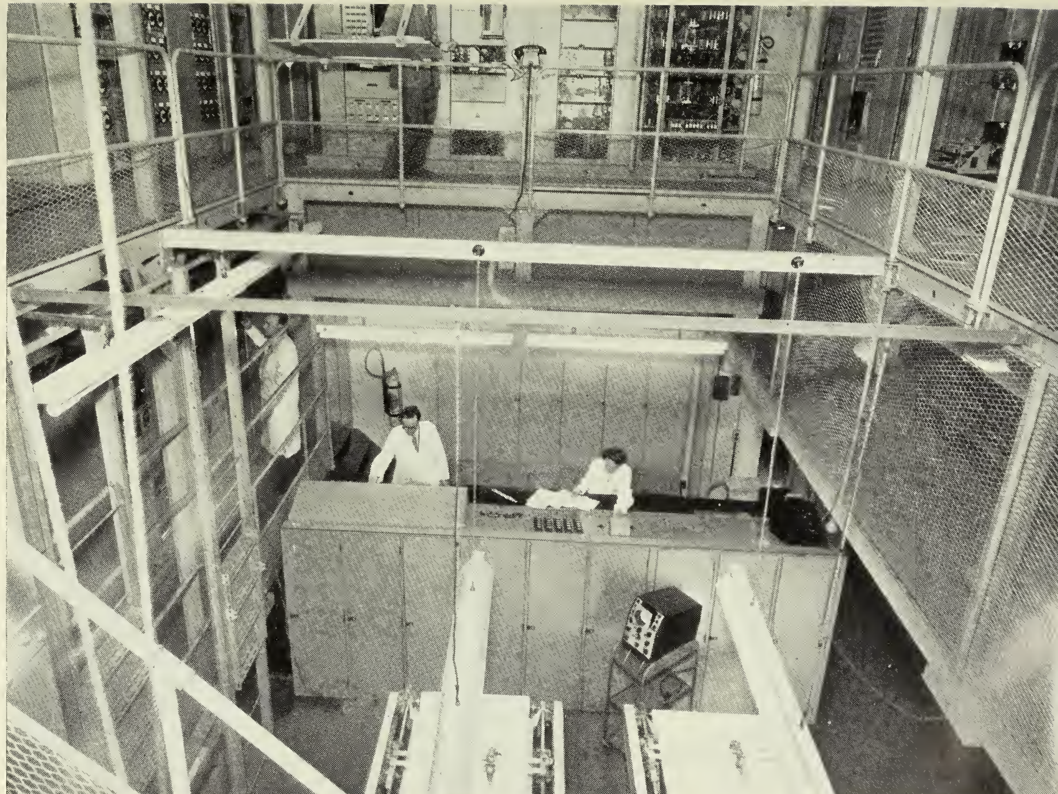

FIG. 141. TRIDAC – Control Room

FIG. 143. TRIDAC – Sine/Cosine Potentiometers

conditions. The peak input power required by the whole machine is about 650 kW. TRIDAC was built by Elliott Bros. (London) Ltd. to a joint R.A.E.-Elliott design.

TRIDAC has not been designed like GEPUS, as a set of basic computing elements of different types which can be connected together as required, but as a set of computing elements arranged to solve a particular class of problems, the elements being arranged in groups such that each group deals with some particular part of the problem. This different approach has naturally affected the relative numbers of the different types of elements and also the layout, but has not appreciably reduced the versatility of the machine. It would be capable, if required, of solving complicated problems of dynamics in other fields besides aircraft flight.

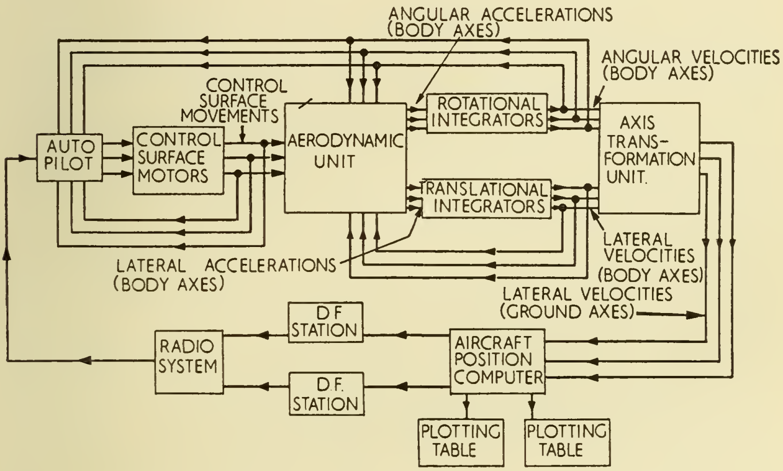

FIG. 142. A Block Diagram for TRIDAC

A typical problem of the type that might be solved on TRIDAC is to determine the behaviour of an aircraft with power-operated rudder, ailerons and elevator, flying under the control of an autopilot which receives correcting signals from a ground station. A number of different schemes are known by which information could be passed from the ground to the aircraft so as to steer it automatically towards some given point. One scheme, for which a block diagram is shown in fig. 142, employs two or more direction-finding stations on the ground to determine the aircraft position relative to ground axes. The data describing this position are sent by a radio system to the aircraft auto-pilot, which has also been fed with data describing the position of the destination point relative to the same set of ground axes. Comparing present position and destination the auto-pilot computes the required movements of the control surfaces to steer the aircraft in the proper direction. In the simulator, the outputs from the simulated auto-pilot are fed into networks representing the lags and non-linearities of the motors which are used to drive the control surfaces, and the voltages representing the actual control surface deflections are fed into the aerodynamic unit, and also back into the auto-pilot. This voltage feedback simulates the feedback which is often used in practice to improve the linearity and other characteristics of the control-surface motors.

The aerodynamic unit of the simulator is a complex assembly of computing elements which when fed with voltages representing the control-surface deflections gives three output voltages representing the components of angular acceleration of the aircraft about its own three principal axes and three other output voltages representing the components of lateral acceleration along the same three axes. The computation involves solution of the force and moment equations for an aircraft in three dimensions, taking into account such non-linearities as the variation of lift coefficient with incidence, and cross-couplings by which an aerodynamic moment about one axis is induced by simultaneous rotation about the two other axes. The effects of centre of pressure shift due to change of incidence or other causes, the shift of centre of gravity and variation of mass and moments of inertia due to fuel consumption, and the effects of variation of height and engine thrust can also be simulated.

The angular and lateral accelerations are passed through integrators to give the corresponding angular and lateral velocities, still measured in axes fixed relative to the body of the aircraft, and some or all of these velocities are fed back to the aerodynamic unit. They may also be fed back into the auto-pilot to help in the calculation of the control surface deflections. All six velocities are fed into an axis transformation unit, which gives outputs representing the components of aircraft velocity in axes fixed relative to the ground. The operation of this unit depends on a straight-forward, though somewhat lengthy application of classical mechanics, and it will not be described in detail. The transformation involves the solution of three equations of the form:

$$\begin{aligned} u_G &= Au + Bv + Cw \\ v_G &= Du + Ev + Fw \\ w_G &= Gu + Hv + Jw \end{aligned}$$

where u_G, v_G, w_G, are the components of velocity in ground axes, and u, v, w, are the components of velocity in aircraft body axes. The quantities A, B and G are given by:

$$\begin{aligned} A &= -\sin Z \sin Z' + \cos Z \cos Y \cos Z' \\ B &= -\cos X \cos Z \sin Z' + \cos Z' (\sin X \sin Y - \cos X \sin Z \cos Y) \\ G &= -\cos Z \sin Y. \end{aligned}$$

The quantities C to F, H and J, are of the same general form as A and B.

The quantities X, Y, Z, and Z' are angles represented by the positions of shafts in the mechanical computing elements of the simulator, the shafts being driven by four high-speed hydraulic motors whose input quantities are voltages representing the attitude and angular velocity of the aircraft.

The components of aircraft velocity in ground axes are fed into the aircraft position computer, which includes three integrators. If the initial conditions for the integrators are set, at $t = 0$, to represent the position of the aircraft at that instant, in ground axes, the integrator output voltages at any subsequent time will represent the instantaneous position of the aircraft. This information is then passed to the simulated auto-pilot, via a simulated radio system if necessary, so that the outer loop of the system is closed.

With such a simulator the behaviour of the automatically-controlled aircraft can be observed in detail, and in particular the effects of changes can be observed. Assuming that a stable system has been devised, either by paper design or a combination of paper design and simulator tests, it is a relatively simple matter to observe the effects of different sets of aerodynamic derivatives, for example, or to change the form of the computation carried out by the auto-pilot. The effects of gusts can be observed by adding appropriate transient voltages to the voltages representing aircraft velocity. If desired the actual motors used to move the control surfaces can be included in place of the simulated motors, and the whole or part of the auto-pilot can similarly be included.

The system could be used to study the behaviour of a human pilot flying "blind" in response to radio signals from the ground by including a man in the simulator loop. The simulator would operate visual displays, or provide audible signals, of the same type as would occur in the real system, and the man would operate flying controls of the normal pattern from which electrical signals would be fed into the simulator.

The electronic elements of TRIDAC are all built on standard-sized chassis, about $12'' \times 8'' \times 3\frac{1}{2}''$, called brick units. Each of these has two multi-way plugs, one for power supplies and one for signals, and the units are carried on shelves in standard cabinets, each position on each shelf having a pair of multi-way sockets which engage with the two plugs. Thus any unit can be withdrawn or replaced in a few seconds. Each cabinet is about 3 ft. $\times$ 2 ft. $\times$ $6\frac{1}{2}$ ft. high, and holds 56 brick units. On the side remote from the brick units there are mountings for the input and feedback resistors, integrator capacitors, potentiometers for setting parameters, and sets of sockets for making connections between the various brick units in the cabinet and also to other cabinets. Each set of four cabinets is mounted on a steel platform and the complete assembly is called a "raft". There are eleven rafts, varying somewhat in detail according to their different functions, and providing altogether space for over 2,000 brick units. All the rafts are provided with forced-draught cooling, with means for adjusting the air flow over each shelf. The 650 high-gain amplifiers with automatic drift correctors account for 1,300 of the brick unit spaces. The remaining spaces are partly occupied by other brick units, such as thermionic stabilizers for the high tension supplies, monitoring and alarm circuits, etc. Some spaces are left empty to allow for future modification and expansion.

The mechanical computing elements include sets of sine and cosine potentiometers driven by high-speed hydraulic motors which are in turn driven from an oil supply at 2,000 lb./sq.in. The mechanisms are of the "swash-plate" type (fig. 143) in which the rotary motion provided by the motors is converted mechanically into sinusoidal motion of a potentiometer wiper, and the potentiometer windings themselves are linear. Each swash-plate mechanism can drive up to twelve sine potentiometers and twelve cosine potentiometers, so that for a given shaft rotation θ twelve separate quantities, represented by "d.c." voltages, can be multiplied by $\sin\theta$, and twelve other quantities can be multiplied by $\cos\theta$, the 24 products appearing as d.c. voltages. The error in performing these computations at very low frequencies is about 0·2%; there is an amplitude error of about 1% and a phase lag of about 4° at 5·0 c/s, but the amplitude response remains approximately flat up to the nominal cut-off frequency of 20 c/s. In fig. 143 the resistance elements have been removed so that the mechanism can be seen.

The motion of the shaft of the hydraulic motor is controlled by an electrically-operated oil valve, and feedback circuits are arranged so that the shaft rotation is accurately proportional to a d.c. input voltage which is applied to an amplifier which operates the oil valve. In some of the mechanical computing elements feedback is provided by an accurate tachometer, which measures the rate of rotation of the hydraulic motor shaft. Thus, with a high gain in the forward path the complete unit acts as an integrator, giving a shaft rotation which is proportional to the time-integral of the input voltage. There are other mechanical computing elements for which the input voltages vary only slowly, so that electric motors instead of hydraulic motors can be used.

Considerable thought has been given in the design of TRIDAC to the problem of reliability, and also to minimizing the labour of operating the machine and analyzing the results. Besides careful design of equipment, generous rating of components, and ageing of valves, reliability is helped by routine checks of all the brick units. By a comprehensive system of testing and recording it is often possible to detect a valve which, while still in usable condition, is getting near the end of its useful life. The action of the automatic drift correctors is monitored, and an alarm is given if the drift voltage of any amplifier moves outside the range ±2mV. As in the case of GEPUS, a large proportion of the different kinds of faults which can occur in an amplifier are shown up by the inability of the automatic equipment to keep the nominal zero within the prescribed limits. The alarm system shows which cabinet and which position holds the faulty element, so that replacement can be effected very quickly.

Apart from a fault in a computing element an analogue computer is liable to give inaccurate results if overloading occurs in any of the elements. To safeguard against this each element of TRIDAC is fitted with an overload indicator, which operates continuously and gives an alarm if the output voltage exceeds the maximum value for satisfactory linearity.

For recording the output voltages of the simulator there are twelve high-speed pen recorders and a Duddell-type oscillograph, besides cathode-ray oscilloscopes with cameras. There are two plotting tables, each 6 ft. $\times$ 3 ft., giving respectively a plan view and a side-on view of the flight of the aircraft. There are also several cathode-ray tube displays and a multi-channel magnetic-tape recorder. The pen recorders and oscillographs are used to give permanent records, but they are not used for all the runs done on the simulator. In the course of any study on a simulator there are always some runs which are not worth recording, and many whose permanent value is doubtful. The general procedure is therefore to observe the important variables during each run on the c.r. tube displays and plotting tables, and to record those which are likely to be of real value. Whenever the pen recorders or oscillographs are not being used the important variables are recorded on the magnetic-tape recorder and permanent records can be made if required at any later time by playing back the tape into a pen recorder. A serial number is recorded on the tape immediately before each simulator run, and the same number is also included in the written record which gives all the details of the simulator set-up, including the block-diagram, parameter values, etc. Thus any required run can easily be found on the magnetic tape. The magnetic tape can also be used to operate the c.r. tube displays and plotting tables, so that any run which has been recorded on the tape can be reproduced later without using the computing sections of the simulator, and without disturbing the simulator set-up.

APPENDIX

"ANALYZER" SOLUTION FOR THE COUPLED MASS-SPRING-FRICTION PROBLEM

It was mentioned in Chapter 2 that an "analyzer" solution was possible for the coupled system of fig. 15, and two somewhat different methods of setting up an analyzer for this purpose will now be described. The equation to be solved is

$$ap^4y + bp^3y + cp^2y + dpy + ey = fp^2x + gpx + hx \qquad (16)$$

where
$$\begin{aligned} a &= m_1m_2 \\ b &= m_1m_2(\mu_1 + \mu_2) \\ c &= m_1k_2 + m_2k_1 + m_2k_2 + m_1m_2\mu_1\mu_2 \\ d &= m_1\mu_1k_2 + m_2\mu_2k_1 + m_2\mu_2k_2 \\ e &= k_1k_2 \\ f &= k_1m_2 \\ g &= k_1m_2\mu_2 \\ h &= k_1k_2 \end{aligned}$$

Of the two methods available the more obvious, though probably the poorer, removes the derivatives of x by integrating the whole equation twice with respect to time. Assuming that the integration constants are zero this gives:

$$ap^2y + bpy + cy + \frac{d}{p}y + \frac{e}{p^2}y = fx + \frac{g}{p}x + \frac{h}{p^2}x \qquad (54)$$

To set up the block diagram, assume as before that a voltage is available representing the highest derivative of y, i.e. p^2y. Then the lower derivatives, from py to y/p^2, are obtained by means of four integrators. The remaining input voltages for the summing amplifier are the three functions of x, and these may be obtained by feeding the x voltage through two integrators. The block diagram (without coefficient multipliers) is shown in fig. 144. This is a possible arrangement, and might be employed if the differential analyzer in use had six or more integrators and there was nothing to be gained by economizing in computer elements. It is, however, possible to re-arrange the block diagram so that the equation (54) can be solved with only four integrators. The first step in the re-arrangement is merely a re-drawing of fig. 144, the object being to show two integrators, numbers 3 and 4, to the left of the summing amplifiers. The

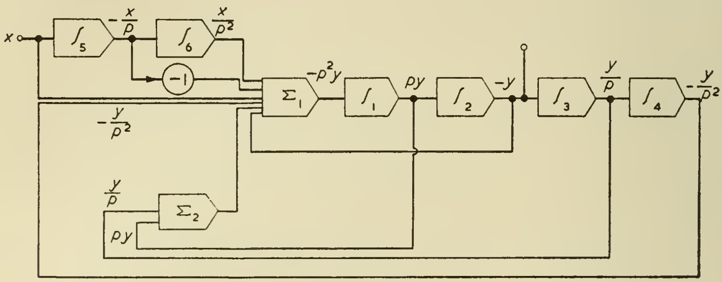

FIG. 144

new arrangement is shown in fig. 145, in which one other small change is shown; the x/p voltage is fed into the second summing amplifier instead of the first so that the reversing amplifier is not needed.

Consider now the two integrators 6 and 4 in fig. 145. Integrator 6 integrates x/p to give x/p^2 and feeds the output to the summing amplifier 1. The same effect can be achieved by omitting integrator 6 and adding a voltage corresponding to x/p to the input voltage to

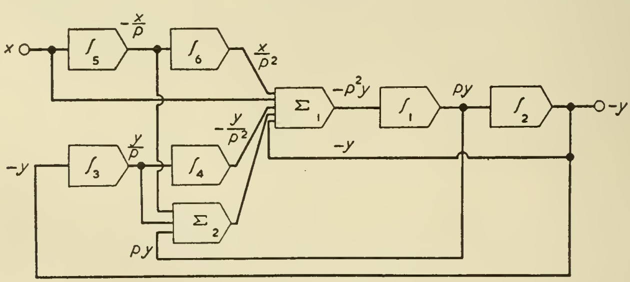

FIG. 145

integrator 4. There is now no input of x/p^2 from the output of integrator 6, but an equal voltage appears as part of the output of integrator 4. The use of an integrator to integrate the sum of two voltages has already been described, and it is, of course, necessary to select a value of the input resistor for the x/p voltage appropriate to the coefficient of x/p. The new arrangement, using only five integrators, is shown in fig. 146.

Integrator 5 is also unnecessary if an appropriate signal is fed to the input of integrator 3, but the situation is complicated by the "feed-forward" from the output of integrator 3 via summing amplifier 2, and by the need to adjust input resistors to allow for the

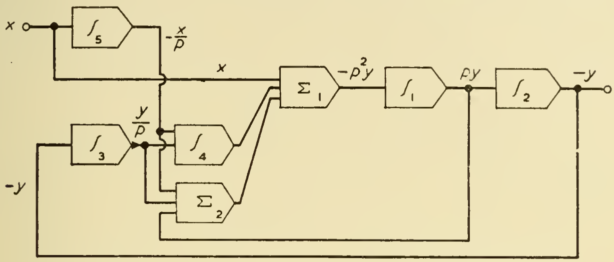

Fig. 146

different coefficients of the x and y terms. Instead of attempting to follow the same procedure as for integrator 6 it is simpler to assume that the arrangement of fig. 147 represents a satisfactory block diagram. Voltages corresponding to x/p are passed to summing amplifier 1 from the output of integrator 4 and also via summing amplifier 2 from integrator 3, and the levels of the x voltages fed into

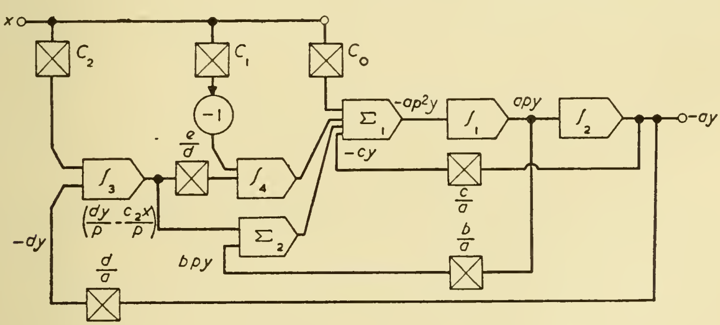

Fig. 147

the integrators must therefore be adjusted to ensure that the total x/p voltage has the correct value. To determine the required adjustment quantitatively assume first that the x voltages being fed into summing amplifier 1 and integrators 4 and 3 are multiplied by factors c_0, c_1 and c_2 respectively. The values of these factors, in terms of the coefficients of the original equation (54) can be determined by writing down the sum of the voltages fed into summing amplifier 1 and equating to p^2y. Before this can be done, however, a consistent set of coefficients must be given to the various functions of y, and this has been done in fig. 147. The necessary changes of scale are indicated by multiplying factors c/a, etc.

Before writing the complete equation it is helpful to write down the total output voltages from integrators 3 and 4. These are, for integrator 3:

$$-\frac{1}{p}(c_2x-dy) = \frac{dy}{p}-\frac{c_2x}{p}$$

and for integrator 4:

$$-\frac{1}{p}\left\{-c_1x-\frac{e}{d}\cdot\frac{1}{p}(c_2x-dy)\right\} = c_1\frac{x}{p}+\frac{e}{d}c_2\frac{x}{p^2}-e\frac{y}{p^2}$$

The full equation is therefore:

$$ap^2y = c_0x+c_1\frac{x}{p}+\frac{e}{d}c_2\frac{x}{p^2}-e\frac{y}{p^2} -d\frac{y}{p} +c_2\frac{x}{p}-bpy-cy$$

or,

$$ap^2y+bpy+cy+d\frac{y}{p}+e\frac{y}{p^2} = c_0x+(c_1+c_2)\frac{x}{p}+\frac{ec_2}{d}\frac{x}{p^2}$$

Comparing with equation (54) gives:

$$f = c_0 \qquad g = c_1+c_2 \qquad h = ec_2/d$$

whence

$$\left.\begin{aligned} c_2 &= hd/e \\ c_1 &= g-hd/e \\ c_0 &= f \end{aligned}\right\} \tag{55}$$

Thus, fig. 147 represents a practical and economical arrangement for solving equation (16).

The arrangement of fig. 147 gives a solution for y only. If a solution is required also for z, this can be obtained by an arrangement like that shown in fig. 11, since by eliminating y from equations (14) and (15) the relation between z and x is found to be:

$$ap^4z+bp^3z+cp^2z+dpz+ez = hx \tag{56}$$

which is of the same form as equation (12).

It has been assumed so far that y and z and all their derivatives were zero at $t=0$. If this is not the case the initial conditions can be inserted by one of the methods of Section 8.2.

A difficulty with this arrangement is that the voltages representing y/p and y/p^2 will readily overload the amplifiers unless both y and y/p have small mean values over the period occupied by the solution.

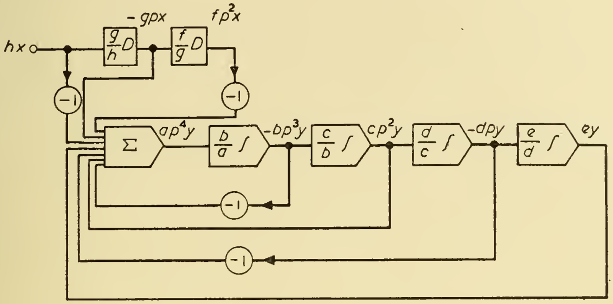

FIG. 148

The difficulty due to the presence of y/p and y/p^2 is removed by using the second method, extended from section 2.3. A block diagram for the solution of equation (16) is first drawn on the assumption that differentiators are available. This can easily be done by the methods given earlier, assuming that a voltage proportional to p^4y is available at the output of a summing amplifier, and the diagram

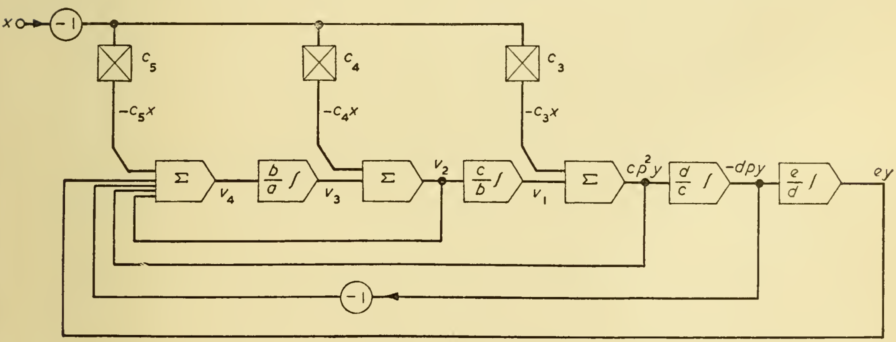

FIG. 149

is shown in fig. 148, where for compactness the coefficient multipliers are now included in the integrator, differentiator and amplifier boxes. In this diagram the differentiated x voltage, $-gpx$, is integrated, together with other voltages, by the b/a integrator; and the voltage

fp^2x, which is the result of two differentiations, is integrated by the b/a and c/b integrators, together with other voltages, and with the further complication due the $-bp^3y$ feedback between the two integrators. However, the discussion given earlier suggests that it would be reasonable to examine the arrangement shown in fig. 149, where the differentiators have been removed, and voltages have been added to the input voltages of the second and third integrators to represent c_4x and c_3x, where c_4 and c_3 are constants whose values are to be determined later. The extra summing amplifiers needed for this purpose have introduced sign changes which make one of the reversing amplifiers unnecessary.

If this arrangement is to give the desired solution it must produce a voltage proportional to y, so it is assumed that the output of the fourth integrator represents ey. It follows that the preceding integrators give output voltages proportional to $-py$ and to p^2y, and it is convenient to label these outputs $-dpy$ and cp^2y. Since the values of c_3, c_4 and c_5 are not yet known the voltages at points earlier than the output of the c/b integrator cannot be determined by inspection, and they are therefore labelled $v_1, \ldots v_4$, as shown.

The following relations can be written down immediately:

$$\begin{aligned} -cp^2y &= v_1 - c_3x \\ pv_1 &= -\frac{c}{b}v_2 \\ -v_2 &= -c_4x + v_3 \\ pv_3 &= -\frac{b}{a}v_4 \\ -v_4 &= -c_5x + ey + dpy + cp^2y + v_2, \end{aligned}$$

and elimination of $v_1, \ldots . v_4$ gives:

$$ap^4y + bp^3y + cp^2y + dpy + ey = \frac{a}{c}c_3p^2x + \left(\frac{a}{b}c_4 + \frac{b}{c}c_3\right)px + c_5x$$

This is identical with equation (16) if

$$\begin{aligned} c_3 &= \frac{c}{a}f \\ c_4 &= \left(\frac{b}{a}g - \frac{b^2}{a^2}f\right) \\ c_5 &= h \end{aligned}$$

so that with suitable adjustments of the values of the resistors which fix the values of these three coefficients this arrangement will give the required solution. In practice the reversing amplifier in fig. 149 could be removed and the $-dpy$ feedback could be fed directly into the b/a integrator.

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SUBJECT INDEX